KD EDUCATION ACADEMY [9582701166]

Time : 7 Hour | Physics | Total Marks : 410

kd 90+ ch-6 system of particles and rotational motion

*** Choose The Right Answer From The Given Options.[1 Marks Each] [44]**

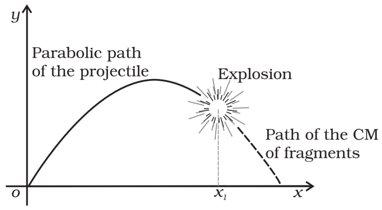
1. A particle performing uniform circular motion has angular momentum L. If its angular frequency is doubled and its kinetic energy halved, then the new angular momentum is:

(A) $4\mathrm{L}$ (B) $\frac{\mathrm{L}}{2}$ (C) $\frac{\mathrm{L}}{4}$ (D) $2\mathrm{L}$

Ans. :

c. $\frac{\mathrm{L}}{4}$

2. A projectile is fired at an angle and it was following a parabolic path. Suddenly, it explodes into fragments. Choose the correct option regarding this situation.



(A) Due to explosion CM shifts upwards.

(B) Due to explosion CM shifts downwards.

(C) Due to explosion CM traces its path back to origin.

(D) CM continues to move along same parabolic path.

Ans. :

c. Due to explosion CM traces its path back to origin.

3. If a girl rotating on a chair bends her hand as shown in figure the (neglecting frictional

force).

(A) I girl will reduce. (B) I girl will increase.

(C) Girl will reduce. (D) None of the above.

Ans. :

a. I girl will reduce.

Explanation:

As there is no external torque, if the girl bends her hands, her MI about the rotational axis will decrease. By conservation of angular momentum,

if $\mathrm{L} = \mathrm{I}\omega$ = constant, then will increase.

4. A wheel of radius 20cm is pushed to move it on a rough horizontal surface. It is found to move through a distance of 60cm on they road during the time it completes one revolution about the centre. Assume that the linear and the angular accelerations are uniform. The frictional force acting on the wheel by the surface is:

(A) Along the velocity of the wheel.

(B) Opposite to the velocity of the wheel.

(C) Perpendicular to the velocity of the wheel.

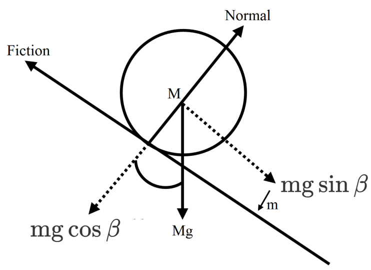
(D) Zero.

Ans. :

a. Along the velocity of the wheel.

Explanation:

As the distance covered in one revolution about the centre is less than the perimeter of the wheel, it means that the direction of torque due to frictional force opposes the motion of wheel, i.e., the frictional force acting on the wheel by the surface is along the velocity of the wheel.



5. A torque of 0.5Nm is required to drive a screw into a wooden frame with the help of a screw driver. If one of the two forces of couple produced by screw driver is 50N, the width of the screw driver is:

(A) 0.5cm (B) 0.75cm (C) 1cm (D) 1.5cm

Ans. :

c. 1cm

Explanation:

Let width be x. So the torque due to couple is 2 × 50 × x/ 2 = 50x

50x = 0.5 ⇒ x = 0.01m = 1cm

6. A body of mass 5kg undergoes a change in speed from 30 to 40m/ s. Its momentum would increase by:

(A) 50kgm/ s (B) 75kgm/ s (C) 150kgm/ s (D) 350kgm/ s

Ans. :

a. 50kgm/ s

Explanation:

Speed is increased by 10m/s so the momentum will change by 50kg m/ s

7. A metallic sphere having mass 2kg is moving with a velocity of 10m/ s. The momentum of the sphere in kg metre/ sec. will be-

(A) $\frac{1}{5}$ (B) 5 (C) 12 (D) 20

Ans. :

d. 20

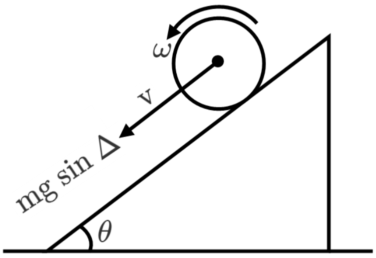
Explanation:

P = m × v

P = 2 × 10

P = 20m/ s

8. Sphere is in pure accelerated rolling motion in the figure shown,



Choose the correct option:

(A) The direction of f, is upwards.

(B) The direction of f, is downwards.

(C) The direction of gravitational force is upwards.

(D) The direction of normal reaction is downwards.

Ans. :

a. The direction of f, is upwards.

9. A bomb travelling in a parabolic path explodes in mid air. The centre of mass of fragments will:

(A) Move vertically upwards and then downwards.

(B) Move vertically downwards.

(C) Move irregularly.

(D) Move in parabolic path, the unexploded bomb would have travelled.

Ans. :

d. Move in parabolic path, the unexploded bomb would have travelled.

Explanation:

On explosion, the centre of mass of all fragments would move in a parabolic path along which unexploded bomb would have moved.

10. If a body of mass M collides against a wall with velocity V and rebounds with the same speed, then its change of momentum will be:

(A) Zero (B) MV (C) 2MV (D) -2MV

Ans. :

c. 2MV

Explanation:

Initial velocity of body before collision u = −V

where minus sign indicates the body moves towards the wall.

Initial momentum of the body $P_i = Mu = -MV$

Final velocity of body after collision v = V

where positive sign indicates the body moves away from the wall.

Final momentum of the body Pf = Mv = MV

11. If there is no external force acting on a nonrigid body, which of the following quantities must remain constant?

(A) Angular momentum. (B) Linear momentum.

(C) Kinetic energy. (D) Moment of inertia.

Ans. :

a. Angular momentum.

b. Linear momentum.

Explanation:

$\overrightarrow{F}_{ext} = 0$

$\Rightarrow \overrightarrow{\tau}_{ext} = 0$

That is, the change in linear momentum and angular momentum is zero. This is because:

$\frac{d\overrightarrow{P}}{dt} = \overrightarrow{F}_{ext}$

And $\frac{d\overrightarrow{L}}{dt} = \overrightarrow{\tau}_{ext}$

12. A flywheel making 120r.p.m is acted upon by a retarding torque producing angular retardation of π rad/ s^2. Time taken by it to come to rest is:

(A) 1s (B) 2s (C) 3s (D) 4s

Ans. :

d. 4s

Explanation:

$T = I\alpha$

$\omega = \frac{120 \times 2\pi}{60}$

$\omega = 4\pi$

$\omega = \omega 0 + \alpha t$

$0 = 4\pi - \pi t$

$t = 4s$

13. A cylindrical solid of mass M has radius R and length L. Its moment of inertia about a generator is:

(A) $M\left(\frac{1}{2R} + \frac{R^2}{4}\right)$ (B) $M\left(\frac{L}{3} + \frac{R^2}{4}\right)$

(C) $\frac{1}{2}MR^2$ (D) $\frac{3}{2}MR^2$

Ans. :

d. $\frac{3}{2}MR^2$

Explanation:

Generator is axis touching surface of cylinder and parallel to axis of cylinder. Using theorem of parallel axis,

$$\mathrm{I} = \mathrm{I}_0 + \mathrm{MR}^2 = \frac{1}{2}\mathrm{MR}^2 + \mathrm{MR}^2$$

$$\mathrm{I} = \frac{3}{2}\mathrm{MR}^2$$

14. A hollow sphere and a solid sphere having same mass and same radii are rolled down a rough inclined plane:

(A) The hollow sphere reaches the bottom first.

(B) The solid sphere reaches the bottom with greater speed.

(C) The solid sphere reaches the bottom with greater kinetic energy.

(D) The two spheres will reach the bottom with same linear momentum.

Ans. :

b. The solid sphere reaches the bottom with greater speed.

Explanation:

Acceleration of a sphere on the incline plane is given by:

$$\mathrm{a} = \frac{\mathrm{g}\sin\theta}{1+\frac{\mathrm{I_{COM}}}{\mathrm{mr}^2}}$$

$\mathrm{I_{COM}}$ for a solid sphere $= \frac{2}{5}\mathrm{mr}^2$

So, $\mathrm{a} = \frac{\mathrm{g}\sin\theta}{1+\frac{2\mathrm{mr}^2}{5\mathrm{mr}^2}} = \frac{5}{7}\mathrm{g}\sin\theta$

$\mathrm{I_{COM}}$ for a hollow sphere $= \frac{2}{3}\mathrm{mr}^2$

So, $\mathrm{a}' = \frac{\mathrm{g}\sin\theta}{1+\frac{2\mathrm{mr}^2}{3\mathrm{mr}^2}} = \frac{3}{5}\mathrm{g}\sin\theta$

The acceleration of the solid sphere is greater; therefore, it will reach the bottom with greater speed.

15. Three objects of same mass but different geometries, capable of rotating have radius of gyration as 0.2, 0.5 and 0.7. A torque is applied to these objects when they are rotating with constant angular velocity. Which object will have a larger response time for showing the change in their angular velocity:

(A) Object with a radius of gyration of 0.7 will take a long time to respond to the torque.

(B) Object with a radius of gyration of 0.5 will take a long time to respond to the torque.

(C) Object with a radius of gyration of 0.2 will take a long time to respond to the torque.

(D) All objects with have same response time.

Ans. :

a. Object with a radius of gyration of 0.7 will take a long time to respond to the torque.

Explanation:

The property of moment of inertia I is a measure of rotational inertia of the body.

Larger the moment of inertia, larger the object resists to the sudden increase or decrease of the speed.

It allows a gradual change in the speed and prevents jerky motions.

16. A body rolls down an inclined plane. If its kinetic energy of rotational motion is 40% of its kinetic energy of translational motion, then the body is a:

(A) Ring.
(B) Cylinder.
(C) Spherical shell.
(D) Solid sphere.

Ans. :

d. Solid sphere.

Explanation:

In case of solid sphere, K.E. of rotation $= \frac{1}{2} \mathrm{I}\, \omega^2$

17. Let $\overrightarrow{\mathrm{A}}$ be a unit vector along the axis of rotation of a purely rotating body and $\overrightarrow{\mathrm{B}}$ be a unit vector along the velocity of a particle P of the body away from the axis. The value of $\overrightarrow{\mathrm{A}} . \overrightarrow{\mathrm{B}}$ is:

(A) 1
(B) -1
(C) 0
(D) None of these.

Ans. :

c. 0

Explanation:

For a purely rotating body, the axis of rotation is always perpendicular to the velocity of the particle. Therefore, we have:

$\overrightarrow{\mathrm{A}} . \overrightarrow{\mathrm{B}} = 0$

18. The radius of gyration of a uniform rod of length L about an axis passing through its centre of mass is:

(A) $\frac{\mathrm{L}}{\sqrt{12}}$
(B) $\frac{\mathrm{L}}{\sqrt{2}}$
(C) $\frac{\mathrm{L}^2}{12}$
(D) $\frac{\mathrm{L}^2}{\sqrt{3}}$

Ans. :

a. $\frac{\mathrm{L}}{\sqrt{12}}$

19. Two spheres of same size one of mass 2kg and another of mass 4kg are dropped simultaneously from the top of Qutab Minar (height = 72m). When they are 1m above the ground, the two spheres have the same:

(A) momentum
(B) kinetic energy
(C) potential energy
(D) acceleration

Ans. :

d. acceleration

Explanation:

Momentum, Kinetic energy and Potential energy has the term of mass in them and since they have different mass but same velocity all these quantity would be different but since acceleration is always 'g' it will be same for both.

20. A body of mass 100g is moving with a velocity of 15m/ s. The momentum associated with the ball will be:

(A) 0.5kg m/ s
(B) 1.5kg m/ s
(C) 2.5kg m/ s
(D) 3.2Ns

Ans. :

b. 1.5kg m/ s

Explanation:

momentum is multiple of mass and velocity so

mass of object in kg is 0.1kg

momentum = 1.5kg m/ s

21. Rolling of ball on the ground is the instance of __________ as well as ____________?

(A) Periodic motion, rotational motion.

(B) Oscillatory motion, rotational motion.

(C) Curvilinear motion, rotational motion.

(D) Rectilinear motion, rotational motion.

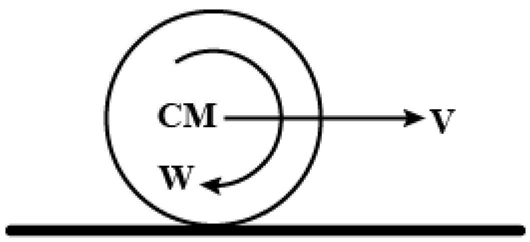
Ans. :

d. Rectilinear motion, rotational motion.

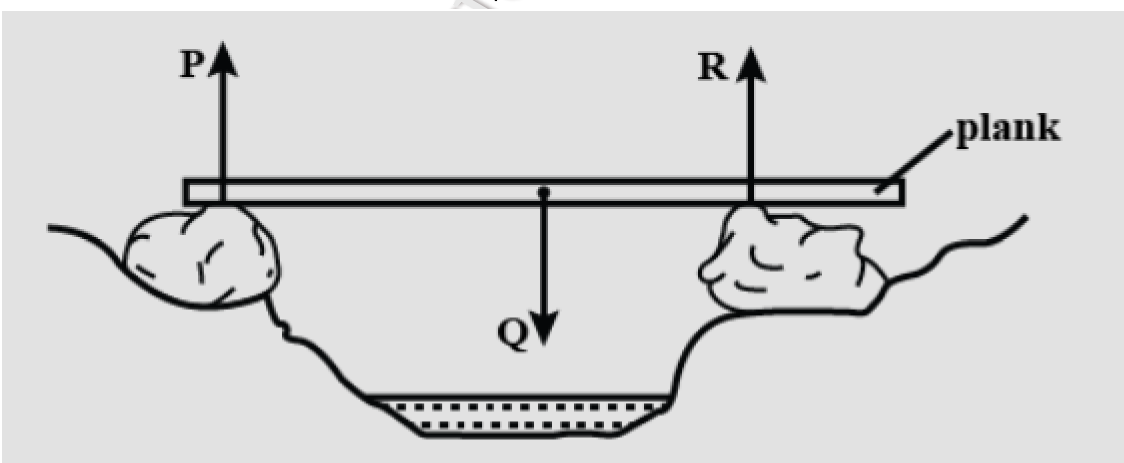
Explanation:

During the rolling of ball on the ground, the center of mass of the ball exhibits translational (rectilinear) motion whereas the ball exhibits rotational motion in center of mass frame.

Thus rolling of ball on the ground is the instance of rectilinear motion as well as rotational motion.



22. A wooden plank rests in equilibrium on two rocks on opposite sides of a narrow stream. Three forces P, Q and R act on the plank. How are the sizes of the forces related?



(A) P + Q = R (B) P + R = Q (C) P = Q = R (D) P = Q + R

Ans. :

b. P + R = Q

Explanation:

A wooden plank rests in equilibrium on two rocks

We know that

In equilibrium the total force acting on the body will be zero.

which means that,

Total force acting upwards = Total force acting upwards

P + R = Q

23. Two particles A and B initially at rest move towards each other under a mutual force of attraction. The speed of centre of mass at the instant when the speed of A is v and the speed of B is 2v is:

(A) V (B) Zero (C) 2v (D) 3v/ 2

Ans. :

b. Zero

Explanation:

The external force acting on particle is zero. so, speed does not not change.

24. The centre of mass of a system particles does not depend on:

(A) Masses of the particles.

(B) Internal forces on the particle.

(C) Position of the particles.

(D) Relative distance between the particles.

Ans. :

b. Internal forces on the particle.

Explanation:

Hint:- Centre of mass depends upon the masses, position and relative distance between the particles.

The resultant of all the internal forces, on any system of particles, is zero.

Therefore their centre of mass does not depend upon the forces acting on the particles.

25. Two balls of different masses have the same KE. Then the:

(A) Heavier ball has greater momentum than the lighter ball.

(B) Lighter Will has greater momentum than the heavier ball.

(C) Both bans have equal momentum.

(D) Both balls have zero momentum.

Ans. :

a. Heavier ball has greater momentum than the lighter ball.

Explanation:

The relation between kinetic energy (K) and momentum (p) is given by,

$\mathrm{K} = \frac{\mathrm{p}^2}{2\mathrm{m}}$ where m be the mass of the body

So, $\mathrm{p} = \sqrt{2\mathrm{mk}}$

As kinetic energy is constant, so $\mathrm{p} \propto \sqrt{\mathrm{m}}$

26. A man of mass M is standing at the centre of a rotating turn table rotating with an angular velocity ω. The man holds two 'dumb bells' of mass $\frac{M}{4}$ each in each of his two hands. If he stretches his arms to a horizontal position, the turn table acquires a new angular velocity ω' where

(A) $\omega' = 2\omega$ (B) $\omega' = \frac{\omega}{2}$ (C) $\omega' > \omega$ (D) $\omega' < \omega$

Ans. :

d. $\omega' < \omega$

27. The theorem of perpendicular axes is applicable for:

(A) Only planar bodies. (B) Only regular shaped bodies.

(C) Only three dimensional bodies. (D) None of the above.

Ans. :

a. Only planar bodies.

Explanation:

Theorem of perpendicular axes is applicable for planar bodies only. We generally use this theorem for regular shaped bodies but it could be applied to irregular shaped bodies as well.

28. The angular velocity of a wheel increases from 100 rps to 300 rps in 10 s. The number of revolutions made during that time is:

(A) 600 (B) 1500 (C) 1000 (D) 2000

Ans. :

d. 2000

Explanation:

Angular displacement θ during time t, assuning constant acccleration be

$\theta = \frac{\omega_0 + \omega_r}{2}$

$= \frac{100+300}{2} \times 10$

$= 2000$ revolutious.

29. The moment of inertia of a uniform semicircular wire of mass M and radius r about a line perpendicular to the plane of the wire through the centre is:

(A) Mr^2 (B) $\frac{1}{2}\mathrm{Mr}^2$ (C) $\frac{1}{4}\mathrm{Mr}^2$ (D) $\frac{2}{5}\mathrm{Mr}^2$

Ans. :

a. Mr^2

Explanation:

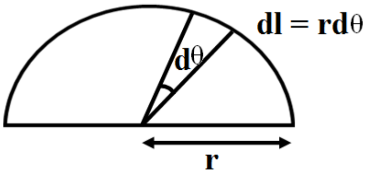
Consider an element of length, $\mathrm{dl} = \mathrm{rd}\theta$

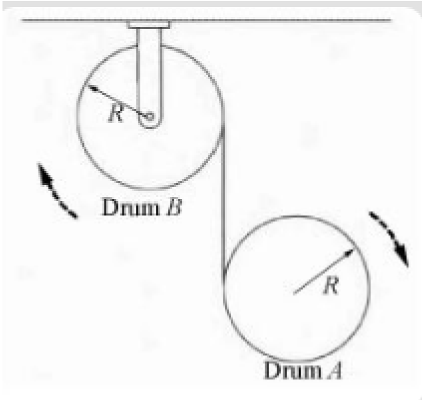
$\mathrm{dm} = \frac{\mathrm{M}}{\pi \mathrm{r}}\mathrm{dl} = \frac{\mathrm{M}}{\pi \mathrm{r}}\mathrm{rd}\theta$

MOI of semicircular wire $= \int_0^\pi \mathrm{r}^2 \mathrm{dm}$

$\mathrm{I} = \int_\pi^0 \mathrm{r}^2 \frac{\mathrm{m}}{\pi \mathrm{r}} \mathrm{rd}\theta$

$\Rightarrow \mathrm{I} = \mathrm{mr}^2$





30. Drum A undergoes:

(A) Rotational motion.

(B) Translational motion.

(C) Rotational as well as translational motion.

(D) None of these.

Ans. :

c. Rotational as well as translational motion.

Explanation:

The ω of drum B provides enough length of connecting wire for translational motion.

31. If the K.E. of a body is increased by 300%, its momentum will increase by _______.

(A) 100% (B) 150% (C) $\sqrt{300}\%$ (D) 175%

Ans. :

a. 100%

Explanation:

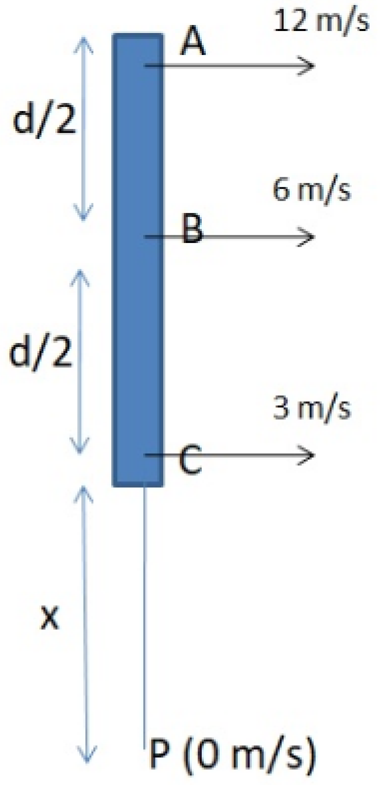
Kinetic energy is given by

K = V^2/ (2M)...............(1)

When KE increases by 300%, the value of KE becomes 4 times it's original value.

from equation (1) that v becomes 2V

32. In the image shown. AC is a rigid rod of length 12cms rotating about point P, whose velocities are shown in the figure. Find the distance PC = x



(A) 12cms (B) 8cms (C) 4cms (D) 2cms

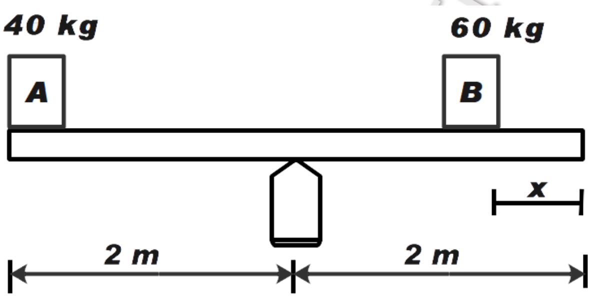
Ans. :

c. 4cms

Explanation:

Since the rod is rotating about the point P, the angular velocities of all the points are same.

33. In the game of see-saw, what should be the displacement of boy B from right edge to keep the see-saw in equilibrium? (M = 40kg, M_2 = 60kg.)



(A) $\frac{4}{3}\mathrm{m}$ (B) $1\mathrm{m}$ (C) $\frac{2}{3}\mathrm{m}$ (D) Zero

Ans. :

c. $\frac{2}{3}\mathrm{m}$

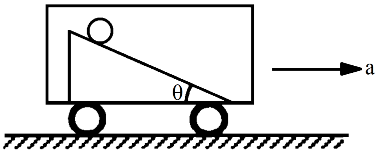
Explanation:

For the equilibrium, $M_1 g \times r_A = M_2 g \times x$ $(40 \times 10) \times 2 = (60 \times 10)x$

$\mathrm{x} = \frac{8}{6} = \frac{4}{3}\mathrm{m}$

So, 60kg boy has to be disptaced $= 2 - \frac{4}{3} = \frac{2}{3}\mathrm{m}.$

34. Figure shows a smooth inclined plane fixed in a car accelerating on a horizontal road. The angle of incline 0 is related to the acceleration a of the car as a = g tan°. If the sphere is set in pure rolling on the incline:

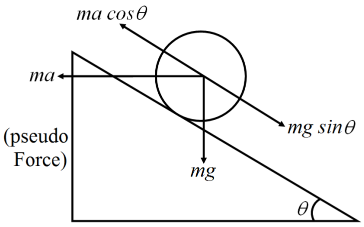


(A) It will continue pure rolling.
(B) It will slip down the plane.
(C) Its linear velocity will increase.
(D) Its linear velocity will slowly decrease.

Ans. :

a. It will continue pure rolling.

Explanation:



From the free body diagram of sphere, we have:

Net force on the sphere along the incline,

$\mathrm{F}_{\text{net}} = \mathrm{mg}\sin\theta - \mathrm{ma}\cos\theta \ \ldots(\mathrm{i})$

On putting $\mathrm{a} = \mathrm{g}\tan\theta$ in equation (i), we get:

$\mathrm{F}_{\text{net}} = 0$

Therefore, if the sphere is set in pure rolling on the incline, it will continue pure rolling.

35. A Merry-go-round, made of a ring-like platform of radius R and mass M, is revolving with angular speed ω A person of mass M is standing on it. At one instant, the person jumps off the round, radially away from the centre of the round (as seen from the round). The speed of the round afterwards is:

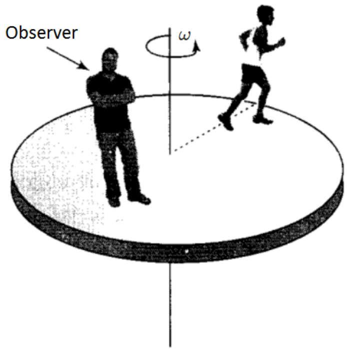
(A) 2ω
(B) ω
(C) $\frac{\omega}{2}$
(D) 0

Ans. :

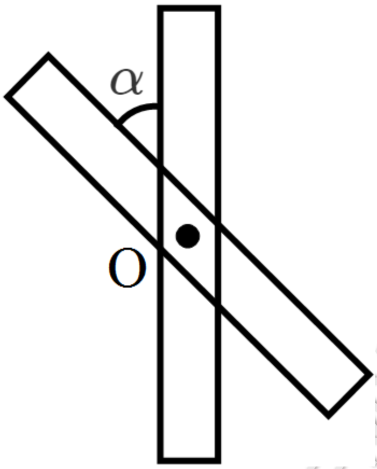
b. ω

Explanation:

As no torque is exerted by the person jumping, radially away from the centre of the round (as seen from the round), let the total moment of inertia of the system is 2I (round + Person (because the total mass is 2M) and the round is revolving with angular speed ω Since the angular momentum of the person when it jumps off the round is $\mathrm{I}\omega$ the actual momentum of round seen from ground is $2\mathrm{I}\omega - \mathrm{I}\omega = \mathrm{I}\omega$ So we conclude that the angular speed remains same, i.e., ω



36. Two identical rods of mass M and length l are lying in a horizontal plane at an angle a. The MI of the system of two rods about an axis passing through O and perpendicular to



the plane of the rods is:

(A) $\frac{Ml^2}{3}$ (B) $\frac{Ml^2}{12}$ (C) $\frac{Ml^2}{4}$ (D) $\frac{Ml^2}{6}$

Ans. :

d. $\frac{Ml^2}{6}$

37. A dancer on ice spins faster when she folds her arms. This is due to:

(A) Increase in energy and increase in angular momentum.

(B) Decrease in friction at the skates.

(C) Constant angular momentum and increase in kinetic energy.

(D) Increase in energy and decrease in angular momentum.

Ans. :

c. Constant angular momentum and increase in kinetic energy.

Explanation:

On folding arms, distance K decreases,

I = MK^2 decreases.

As I × ω = constant,

ω increases

As. K.E. of rotation $\frac{1}{2}I\omega^2$

∴ Due to decrease in I and increase in w, there is overall increase in K.E.

38. A sphere can roll on a surface inclined at an angle θ if the friction coefficient is more than $\frac{2}{7}g\tan\theta$. Suppose the friction coefficient is $\frac{1}{7}g\tan\theta$. If a sphere is released from rest on the incline:

(A) It will stay at rest.

(B) It will make pure translational motion.

(C) It will translate and rotate about the centre.

(D) The angular momentum of the sphere about its centre will remain constant.

Ans. :

c. It will translate and rotate about the centre.

Explanation:

The given coefficient of friction $\left(\frac{1}{7}\mathrm{g}\tan\theta\right)$ is less than the coefficient friction $\left(\frac{2}{7}\mathrm{g}\tan\theta\right)$ required for perfect rolling of the sphere on the inclined plane.

Therefore, sphere may slip while rolling and it will translate and rotate about the centre.

39. A constant torque acting on a uniform circular wheel changes its angular momentum from L to 4L in 4 seconds. The magnitude of this torque is:

(A) $\frac{3\mathrm{L}}{4}$ (B) 4L (C) L (D) 12L

Ans. :

a. $\frac{3\mathrm{L}}{4}$

40. A body of mass 1kg is thrown with a velocity of $10\mathrm{ms}^{-1}$ at an angle of 60° with the horizontal. Its momentum at the highest point is:

(A) $2\mathrm{kg\ ms}^{-1}$ (B) $3\mathrm{kg\ ms}^{-1}$ (C) $4\mathrm{kg\ ms}^{-1}$ (D) $5\mathrm{kg\ ms}^{-1}$

Ans. :

d. $5\mathrm{kg\ ms}^{-1}$

Explanation:

At the highest point, its velocity in the upward direction is zero but its velocity in the horizontal direction remains constant at every instant.

Hence , only the horizontal component of velocity is responsible for the momentum of the body at the highest point.

Velocity of the body $v = 10\mathrm{ms}^{-1}$

Velocity component of the body in the horizontal direction

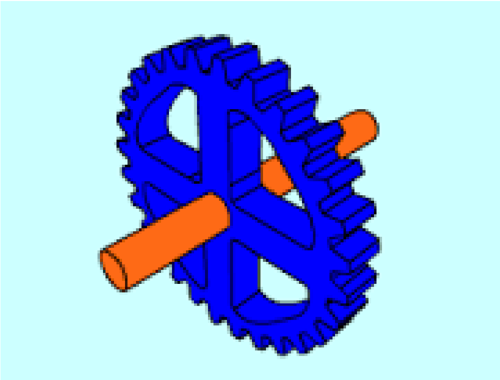
$v_H = v\cos 60° = 10 \times 0.5 = 5\ \mathrm{ms}^{-1}$

$\therefore$ Momentum $P = mv_H = 1 \times 5 = 5\mathrm{kg\ ms}^{-1}$

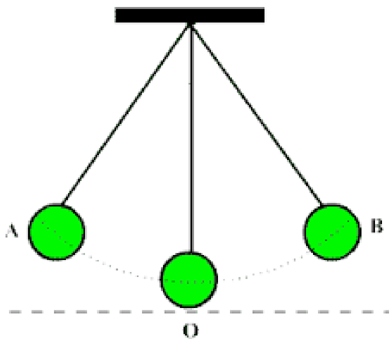
41. Which of the following doesn't represent rotatory motion?

(A)

(B)



(C)



(D) Both A and C

Ans. :

d. Both A and C

Explanation:

In picture A, the running people are exhibiting translational motion. In picture B, the wheel rotates about a fixed axis and thus it exhibits rotatory motion whereas in picture C, the bob of pendulum oscillates to and fro about its mean position and thus it exhibits oscillatory motion.

42. A particle of mass Mkg describes a circle of radius 1m. The centripetal acceleration of the particle is $4 m/s^2$. What will be the momentum of the particle ?

(A) 4M (B) 2M (C) 8M (D) M

Ans. :

b. 2M

Explanation:

Centripetal acc. = v^2/r

$4 = v^2 \Rightarrow |v| = 2$

Now, momemntum = Mv = 2M

43. A person sitting firmly over a rotating stool has his arms stretched. If he folds his arms, his angular momentum about the axis of rotation:

(A) Increases. (B) Decreases.

(C) Remains unchanged. (D) Doubles.

Ans. :

c. Remains unchanged.

Explanation:

Rate of change of angular momentum of the body is directly proportional to the net external torque acting on the body.

No external torque is applied on the person or on the table; therefore, the angular momentum will be conserved.

44. What is the moment of inertia of a ring about a tangent to the periphery of the ring?

(A) $\frac{1}{2}\mathrm{MR}^2$ (B) $\frac{3}{2}\mathrm{MR}^2$ (C) MR^2 (D) $\mathrm{MR}^2\,\frac{2}{9}$

Ans. :

b. $\frac{3}{2}\mathrm{MR}^2$

* Answer The Following Questions In One Sentence.[1 Marks Each] [8]

45. A rifle barrel has a spiral groove which imparts spin to the bullet. Why?

Ans. : Angular momentum gained by the bullet provides better accuracy.

46. What type of motion is produced by couple?

Ans. : Only rotational motion.

47. Why do we place handles at maximum possible distance from the hinges in a door?

Ans. : To develop torque with less force being applied.

48. What is the moment of inertia of a sphere of mass 20kg and radius m about its diameter?

Ans. : Moment of inertia of a sphere about its diameter,

$$\mathrm{I}=\frac{2}{5}\mathrm{mR}^2=\frac{2}{5}\times\left(\frac{1}{4}\right)^2$$
$$=0.5\mathrm{kg}-\mathrm{m}^2$$

49. If the ice on the polar caps of the earth melts, how will it affect the duration of the day? Explain.

Ans. : Earth rotates about its polar axis. When ice of polar caps of earth melts, mass concentrated near the axis of rotation spreads out. Therefore, moment of inertia I increases.

As no external torque acts,

$\therefore \mathrm{L}=\mathrm{I}\omega=\mathrm{I}\left(\frac{2\pi}{\mathrm{T}}\right)$ = Constant. With increase of I, T will increase, i.e., length of the day will increase.

50. Find the torque of a force $7\hat{\mathrm{i}} - 3\hat{\mathrm{j}} - 5\mathrm{k}$ about the origin which acts on a particle whose position vector is $\hat{\mathrm{i}} + \hat{\mathrm{j}} - \hat{\mathrm{E}}$.

Ans. : $\vec{\mathrm{F}} = 7\hat{\mathrm{i}} - 3\hat{\mathrm{j}} - 5\hat{\mathrm{k}}$

$\vec{\mathrm{r}} = \hat{\mathrm{r}} + \hat{\mathrm{j}} - \hat{\mathrm{k}}$

$\vec{\tau} = \vec{\mathrm{r}} \times \vec{\mathrm{F}} = (\hat{\mathrm{i}} + \hat{\mathrm{j}} - \hat{\mathrm{k}}) \times (7\hat{\mathrm{i}} - 3\hat{\mathrm{j}} - 5\hat{\mathrm{k}})$

$= -8\hat{\mathrm{i}} - 2\hat{\mathrm{j}} - 10\hat{\mathrm{k}}$

51. How will you distinguish between a hard boiled egg and a raw egg by spinning each on a table top?

Ans. : To distinguish between a hard boiled egg and a raw egg, we spin each on a table top. The egg which spins at a slower rate shall be a raw egg. This is because in a raw egg, liquid matter inside tries to get away from the axis of rotation. Therefore, its moment of inertia I increases. As $\tau = \mathrm{I}\ \alpha =$ constant, therefore, a decreases, i.e., raw egg will spin with smaller angular acceleration. The reverse is true for a hard boiled egg which will rotate more or less like a rigid body.

52. Explain how a cat is able to land on its feet after a fall taking advantage of the principle of conservation of angular momentum.

Ans. : While falling a cat stretches its body along with the tail so that its moment of inertia (I) increases. As no external torque acts, L = Iw = constant. As I increases, w decreases and it lands gently on its feet.

*** Given Section consists of questions of 2 marks each.** **[30]**

53. Find the scalar and vector products of two vectors. $a = (3\hat{i} - 4\hat{j} + 5\hat{k})$ and $b = (-2\hat{i} + \hat{j} + 3\hat{k})$

Ans. :

$$\begin{aligned} a \cdot b &= (3\hat{i} - 4\hat{j} + 5\hat{k}) \cdot (-2\hat{i} + \hat{j} - 3\hat{k}) \\ &= -6 - 4 - 15 \\ &= -25 \end{aligned}$$

$$a \times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 5 \\ -2 & 1 & -3 \end{vmatrix} = 7\hat{i} - \hat{j} - 5\hat{k}$$

Note $b \times a = -7\hat{i} + \hat{j} + 5\hat{k}$

54. Find the torque of a force $7\tilde{i} + 3\bar{j} - 5\tilde{k}$ about the origin. The force acts on a particle whose position vector is $\tilde{i} - \tilde{j} + \tilde{k}$.

Ans. : Here $r = \hat{i} - \hat{j} + \hat{k}$

and $F = 7\hat{i} + 3\hat{j} - 5\hat{k}$.

We shall use the determinant rule to find the torque $\tau = r \times F$

$$\tau \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 7 & 3 & -5 \end{vmatrix} = (5-3)\hat{i} - (-5-7)\hat{j} + (3-(-7))\hat{k}$$

or $\tau = 2\hat{i} + 12\hat{j} + 10\hat{k}$

55. Give the location of the centre of mass of a:

i. Sphere.
ii. Cylinder.
iii. Ring, and
iv. Cube, each of uniform mass density.

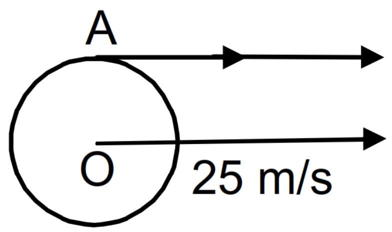
Does the centre of mass of a body necessarily lie inside the body?

Ans. : In all the four cases, as the mass density is uniform, the centre of mass is located at their respective geometrical centres. No, it is not necessary that the centre of mass of a body should lie on the body.

For example: in the case of a circular ring, centre of mass is at the centre of the ring, where there is no mass.

56. A cylinder rolls on a horizontal plane surface. If the speed of the centre is 25m/s, what is the speed of the highest point?

Ans. : A cylinder rolls in a horizontal plane having centre velocity 25m/s.



At its age the velocity is due to its rotation as well as due to its leniar motion & this two velocities are same and acts in the same direction (v = rω)

Therefore Net velocity at A = 25m/s + 25m/s = 50m/s

57. A ring, a disc and a sphere all of the same radius and same mass roll down on an inclined plane from the same height h. Which of the three reaches the bottom (i) earliest (ii) latest?

Ans. : We have already deduced that acceleration of an object down on inclined plane is given by

$$\alpha = \frac{\mathrm{g}\sin\theta}{\mathrm{I}+\left(\frac{\mathrm{I}}{\mathrm{mr}}\right)^2}$$

For a ring $\mathrm{I} = \mathrm{mr}^2$

$\therefore \mathrm{a_{ring}} = \frac{\mathrm{g}\sin\theta}{1+1} = 0.5\mathrm{g}\sin\theta$

For a disc, $\mathrm{I} = \frac{1}{2}\mathrm{mr}^2$

$\therefore \mathrm{a_{disc}} = \frac{\mathrm{g}\sin\theta}{1+\frac{1}{2}} = \frac{2}{3}\mathrm{g}\sin\theta$

$= 0.67\mathrm{g}\sin\theta$

For a sphere, $\mathrm{I}=\frac{2}{5}\mathrm{mr}^2$

$\therefore \mathrm{a}_{\text{sphere}}=\frac{\mathrm{g}\sin\theta}{1+\frac{2}{5}}=\frac{5}{7}\mathrm{g}\sin\theta=0.71\mathrm{g}\sin\theta.$

As $\mathrm{a}_{\text{sphere}}$ is maximum, it will reach the bottom at the earliest. Again as a_{ring} is minimum, it will reach the bottom at the end.

58. If earth contracts to half its radius, what would be the duration of the day? Ans. According to the law of conservation of angular momentum,

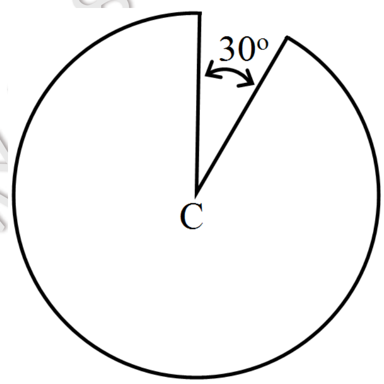
Ans. : $\mathrm{I}_1\omega_1=\mathrm{I}_2\omega_2$

$\Rightarrow \frac{\mathrm{I}_1}{\mathrm{T}_1}=\frac{\mathrm{I}_2}{\mathrm{T}_2}$ or $\mathrm{T}_2=\frac{\mathrm{I}_2}{\mathrm{I}_1}\mathrm{T}_1$

$\mathrm{I}_1=\frac{2}{5}\mathrm{MR}^2, \mathrm{I}_2=\frac{2}{5}\mathrm{M}\left(\frac{\mathrm{R}}{2}\right)^2$

$\therefore \mathrm{T}_1=\frac{1}{4}\times 24=6\mathrm{hr}.$

59. From a complete ring of mass M and radius R, a 30° sector is removed. What is the moment of inertia of the incomplete ring about an axis passing through the centre of



the ring and perpendicular to the plane of the ring?

Ans. : Mass of incomplete ring $=\mathrm{M}-\frac{\mathrm{M}}{2\pi}\times\frac{\pi}{6}$

$=\mathrm{M}-\frac{\mathrm{M}}{2}=\frac{11}{12}\mathrm{M}$

Moment of inertia of incomplete ring

$=\left(\frac{11\mathrm{M}}{2}\right)\mathrm{R}^2=\frac{11}{12}\mathrm{MR}^2$

60. Two bodies of masses 1kg and 2kg are located at (1, 2) and (-1, 3) respectively. Calculate the coordinates of the centre of mass.

Ans. : $\mathrm{X}_{\mathrm{cm}}=\frac{\mathrm{m}_1\mathrm{x}_1+\mathrm{m}_2\mathrm{x}_2}{\mathrm{m}_1+\mathrm{m}_2}$

$=\frac{1\times1+2\times(-1)}{3}=\frac{-1}{3}$

$\mathrm{Y}_{\mathrm{cm}}=\frac{\mathrm{m}_1\mathrm{y}_1+\mathrm{m}_2\mathrm{y}_2}{\mathrm{m}_1+\mathrm{m}_2}$

$=\frac{1\times2+2\times3}{3}=\frac{9}{3}=\frac{8}{3}$

Coordinate of centre of mass (COM) is $\left(\frac{-1}{3},\frac{8}{3}\right)$

61. Why does a girl have to lean towards right when carrying a bag in her left hand?

Ans. : When the girl carries a bag in her left hand, the centre of gravity of the system is shifted to the left. In order to bring it in the middle, the girl has to lean towards right.

62. A boy is seated in a revolving chair revolving at an angular speed of 120 revolutions per minute. Two heavy balls form part of the revolving system and the boy can pull the balls closer to himself or may push them apart. If by pulling the balls closer, the boy decreases the moment of inertia of the system from 6kg-m^2 to 2kg-m^2, what will be the new angular speed?

Ans. : $\omega = 120\text{rpm} = 120 \times \left(\frac{2\pi}{60}\right) = 4\pi\text{rad/s}$

$\mathrm{l}_1 = 6\text{kg-m}^2,\ \mathrm{l}_2 = 2\text{Kg-m}^2$

Since two balls are inside the system

Therefore, total external torque = 0

Therefore $\mathrm{l}_1\omega_1 = \mathrm{l}_2\omega_2$

$\Rightarrow 6 \times 4\pi = 2\omega_2$

$\Rightarrow \omega_2 = 12\pi\text{rad/s}$

$= 6\text{rev/s}$

$= 360\text{rev/ minute.}$

63. A wheel is making revolutions about its axis with uniform angular acceleration. Starting from rest, it reaches 100rev/sec in 4 seconds. Find the angular acceleration. Find the angle rotated during these four seconds.

Ans. : $\omega_0 = 0;\ \rho = 100\text{rev/s};\ \omega = 2\pi;\ \rho = 200\pi\text{rad/s}$

$\Rightarrow \omega = \omega_0 = \alpha\text{t}$

$\Rightarrow \omega = \alpha\text{t}$

$\Rightarrow \alpha = \left(\frac{200\pi}{4}\right) = 50\pi\text{rad/s}^2$ or 25rev/s^2

$\therefore \theta = \omega_0\text{t} + \frac{1}{2}\alpha\text{t}^2 = 8 \times 50\pi$

$= 400\pi\text{rad}$

$\therefore \alpha = 50\pi\text{rad/s}^2$ or 25rev/s^2

$\theta = 400\pi\text{rad}$

64. Find the torque of a force $(7\hat{\mathrm{i}} + 3\hat{\mathrm{j}} - 5\hat{\mathrm{k}})$ N about the origin, the force acts on a particle whose position vector is $(7\hat{\mathrm{i}} - \hat{\mathrm{j}} + \hat{\mathrm{k}})\mathrm{m}$.

Ans. : $\left.\begin{array}{l} \mathrm{F} = 7\hat{\mathrm{i}} + 3\hat{\mathrm{j}} - 5\hat{\mathrm{k}}\ \mathrm{N} \\ \text{Position vector } \vec{\mathrm{r}} = \hat{\mathrm{i}} - \hat{\mathrm{j}} + \hat{\mathrm{k}}\ \mathrm{m} \end{array}\right]\text{(Given)}$

The torque or moment of a force is given by

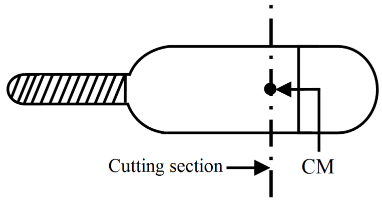
$\vec{\tau} = \vec{\mathrm{r}} \times \vec{\mathrm{F}}$

$\vec{\tau}\begin{vmatrix} \hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}} \\ 1 & -1 & 1 \\ 7 & 3 & -3 \end{vmatrix}$

$\vec{\tau} = \hat{\mathrm{i}}(5-3) - \hat{\mathrm{j}}(5-7) + \hat{\mathrm{k}}(3+7)$

$\vec{\tau} = 2\hat{\mathrm{i}} + 12\hat{\mathrm{j}} + 10\hat{\mathrm{k}}$

65. A cricket bat is cut through its centre of mass into two parts as shown



Then, state whether both parts are of same mass or not. Also, give reason.

Ans. : Centre of mass of a body lies towards region of heavier mass. So, if bat is cut through its centre of mass, both parts are not of equal masses.

66. A car is moving on road with speed 54kmh. What should be the value of torque if the car is brought to rest in 15 seconds? Radius and moment of inertia of wheel about the axis of rotation are 0.35m and 3kgm respectively.

Ans. : Angular acceleration $\alpha = \frac{\omega_t - \omega_0}{t}$

$= \frac{0 - \frac{15}{0.35}}{15} = \frac{-1}{0.35} \text{rad s}^{-2}$

Torque $\tau = I\alpha = 3 \times \left(\frac{-1}{0.35}\right) \text{kg m}^2 \text{s}^{-2}$

$= -87 \text{kg m}^2 \text{s}^{-2}$

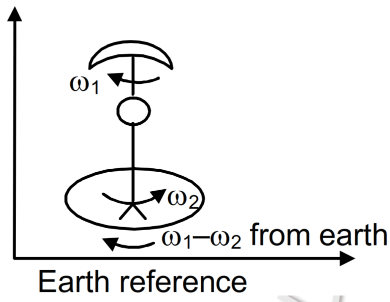
Speed of car = 54kmh^{-1} = 15ms^{-1}

Radius of a car = 0.35m

Angular velocity $\omega_0 = \frac{\text{speed}}{R} = \frac{15}{0.35}, \text{Initial } \omega_t = 0$

67. A boy is standing on a platform which is free to rotate about its axis. The boy holds an open umbrella in his hand. The axis of the umbrella coincides with that of the platform. The moment of inertia of ''the platform plus the boy system'' is 3.0×10^{-3}kg-m^2 and that of the umbrella is 2.0×10^3kg-m^2. The boy starts spinning the umbrella about the axis at an angular speed of 2.0rev/s with respect to himself. Find the angular velocity imparted to the platform.

Ans. :



$l_1 = 2 \times 10^{-3} \text{Kg-m}^2$

$l_2 = 3 \times 10^{-3} \text{Kg-m}^2$

$\omega_1 = 2 \text{rad/s}$

From the earth reference the umbrella has a angular velocity $(\omega_1 - \omega_2)$

And the angular velocity of the man will be ω_2

Therefore $l_1(\omega_1 - \omega_2) = l_2\omega_2$

$\Rightarrow 2 \times 10^{-3}(2 - \omega_2) = 3 \times 10^{-3} \times \omega_2$

$\Rightarrow 5\omega_2 = 4$

$\Rightarrow \omega_2 = 0.8\text{rad/s}.$

*** Given Section consists of questions of 3 marks each. [111]**

68. A bullet of mass 10g and speed 500m/s is fired into a door and gets embedded exactly at the centre of the door. The door is 1.0m wide and weighs 12kg. It is hinged at one end and rotates about a vertical axis practically without friction. Find the angular speed of the door just after the bullet embeds into it. (**Hint:** The moment of inertia of the door about the vertical axis at one end is $ML^2/3$).

Ans. : Angular momentum imparted by the bullet

$\mathrm{L} = \mathrm{mv} \times \mathrm{r}$

$= (10 \times 10^{-3}) \times 500 \times \frac{1}{2}$

$= 2.5$

Also, $\mathrm{I} = \frac{\mathrm{Mb}^2}{3}$

$= \frac{12 \times 1.0^2}{3}$

$= 4\mathrm{kg.m}^2$

As $\mathrm{L} = \mathrm{I}\omega$

$\therefore\ \omega = \frac{\mathrm{L}}{\mathrm{I}} = \frac{2.5}{4} = 0.625\text{rad/sec}$

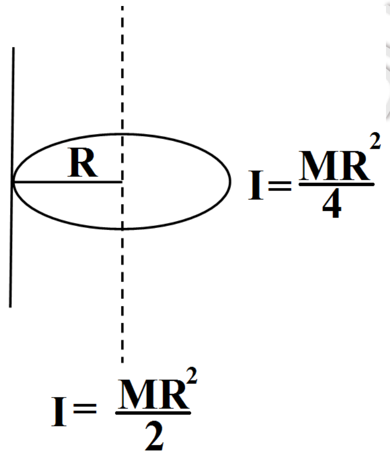
69. Given the moment of inertia of a disc of mass M and radius R about any of its diameters to be $\frac{\mathrm{MR}^2}{4}$, find its moment of inertia about an axis normal to the disc and passing through a point on its edge.

Ans. : The moment of inertia of a disc about its diameter $= \frac{\mathrm{MR}^2}{4}$

According to the theorem of perpendicular axis, the moment of inertia of a planar body (lamina) about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes concurrent with perpendicular axis and lying in the plane of the body.

The M.I. of the disc about its centre $= \frac{\mathrm{MR}^2}{4} + \frac{\mathrm{MR}^2}{4} = \frac{\mathrm{MR}^2}{2}$

The situation is shown in the given figure:



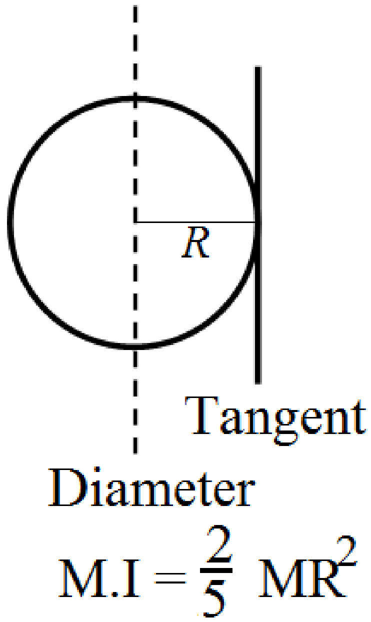
Applying the theorem of parallel axes,

The moment of inertia about an axis normal to the disc and passing through a point on its edge

$= \frac{MR^2}{2} + MR^2 = \frac{3MR^2}{2}$

70. Find the moment of inertia of a sphere about a tangent to the sphere, given the moment of inertia of the sphere about any of its diameters to be $2MR^2/5$, where M is the mass of the sphere and R is the radius of the sphere.

Ans. : The moment of inertia (M.I.) of a sphere about its diameter $= \frac{2}{5}MR^2$



According to the theorem of parallel axes, the moment of inertia of a body about any axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its centre of mass and the product of its mass and the square of the distance between the two parallel axes.

The M.I. about a tangent of the sphere $= \frac{2MR^2}{5} + MR^2 = \frac{7MR^2}{5}$

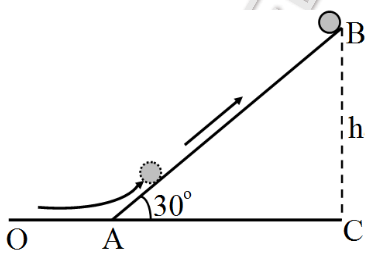
71. A solid cylinder rolls up an inclined plane of angle of inclination 30°. At the bottom of the inclined plane the centre of mass of the cylinder has a speed of 5m/s.

a. How far will the cylinder go up the plane?

b. How long will it take to return to the bottom?

Ans. :

a. A solid cylinder rolling up an inclination is shown in the following figure:



Initial velocity of the solid cylinder, v = 5m/s

Angle of inclination, $\theta = 30^\circ$

Height reached by the cylinder = h Energy of the cylinder at point A

$\mathrm{KE_{root}} = \mathrm{KE_{trans}}$

$\frac{1}{2}\mathrm{I}\omega^2 = \frac{1}{2}\mathrm{mv}^2$

Energy of the cylinder at point B = mgh

Using the law of conservation of energy, we can write,

$\frac{1}{2}\mathrm{I}\omega^2 = \frac{1}{2}\mathrm{mv}^2 = \mathrm{mgh}$

Moment of inertia of the solid cylinder, $\mathrm{I} = \frac{1}{2}\mathrm{mr}^2$

$\therefore\ \frac{1}{2}\left(\frac{1}{2}\mathrm{mr}^2\right)\omega^2 + \frac{1}{2}\mathrm{mv}^2 = \mathrm{mgh}$

$\frac{1}{4}\mathrm{mr}^2\omega^2 + \frac{1}{2}\mathrm{mv}^2 = \mathrm{mgh}$

But we have the relation, $\mathrm{v} = \mathrm{r}\omega$

$\therefore\ \frac{1}{4}\mathrm{v}^2 + \frac{1}{2}\mathrm{v}^2 = \mathrm{gh}$

$\frac{3}{4}\mathrm{v}^2 = \mathrm{gh}$

$\therefore\ \mathrm{h} = \frac{3}{4}\frac{\mathrm{v}^2}{\mathrm{g}}$

$= \frac{3}{4} \times \frac{5\times5}{9.8} = 1.91\mathrm{m}$

b. In $\Delta\mathrm{ABC}$

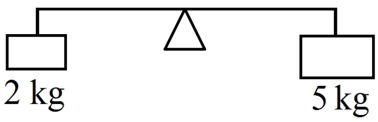
$\sin\theta = \frac{\mathrm{BC}}{\mathrm{AB}}$

$\sin 30^\circ = \frac{\mathrm{h}}{\mathrm{AB}}$

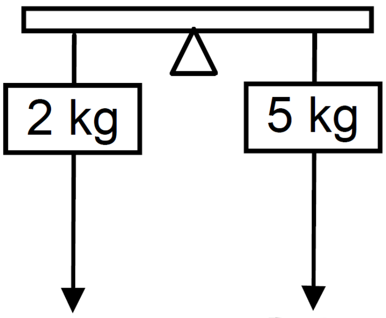
$\mathrm{AB} = \frac{1.91}{0.5} = 3.82\mathrm{m}$

Hence, the cylinder will travel 3.82m up the inclined plane. For radius of gyration K, the velocity of the cylinder at the instance when it rolls back to the bottom is given by the relation.

72. A light rod of length 1m is pivoted at its centre and two masses of 5kg and 2kg are hung from the ends as shown in figure. Find the initial angular acceleration of the rod assuming that it was horizontal in the beginning.



Ans. :



$\mathrm{I}_{\text{net}} = \mathrm{I}_{\text{net}} \times \alpha$

$\Rightarrow \mathrm{F}_1\mathrm{r}_1 - \mathrm{F}_2\mathrm{r}_2 = \left(\mathrm{m}_1\mathrm{r}_1^2 + \mathrm{m}_2\mathrm{r}_2^2\right) \times \alpha - 2 \times 10 \times 0.5$

$\Rightarrow 5 \times 10 \times 0.5 = \left(5 \times \left(\frac{1}{2}\right)^2 + 2 \times \left(\frac{1}{2}\right)^2\right) \times \alpha$

$\Rightarrow 15 = \frac{7}{4}\alpha$

$\Rightarrow \alpha = \frac{60}{7} = 8.57\mathrm{rad/s}^2$

73.

a. Why do you prefer to use a wrench of long arm?

b. A 3m long ladder weighing 20kg leans on a frictionless wall. Its feet rest on the floor 1m from the wall. Find the reaction forces of the wall and the floor.

Ans. :

a. The turning effect of force $\vec{\tau} = \vec{r} \times \vec{F}$ when $|\vec{r}|$ when is large, smaller force F will produce the same turning effect.

b. Force on ladder:

i. W vertically downward at D.

ii. F_1, i.e. reaction of wall perpendicular $(\perp)$ to the wall as well as wall is frictionless.

iii. Reaction of the floor F_2, at an angle θ with floor.

It can be resolved into rectangular components as N and F.

For transitional equilibrium in vertical direction

W = 0

$\therefore$ N = W = 20kg = 20 × 9.8 = 196 N

For transitional equilibrium in horizontal direction,

$F - F_1 = 0$

$\Rightarrow F_1 = F$

For rotational equilibrium,

$$F_1 \times 2\sqrt{2} - W\left(\frac{1}{2}\right) = 0$$

$$F_1 = \frac{W}{4\sqrt{2}} = \frac{196}{4\sqrt{2}} = 34.64 \text{ N} = F$$

$$F_2 = \sqrt{N^2 + F^2}$$

$$= \sqrt{(196)^2 + (34.64)^2} = 199.0 \text{ N}$$

$$\because \tan\theta = \frac{N}{F} = \frac{196}{34.64} = 5.65$$

$$\Rightarrow \theta = \tan^{-1}(5.65) = 79.96$$

$$\therefore \theta \approx 80^\circ$$

74. A particle starts rotating from rest and displaces according to the formula. $\theta = \frac{3t^3}{20} - \frac{t^2}{3}$ Calculate the angular velocity and angular acceleration at the end of 5sec.

Ans. : $\omega = \frac{d}{dt}\left[\frac{3t^3}{20} - \frac{t^2}{3}\right]$

$$= \frac{3}{20} \times 3t^2 - \frac{1}{2} \times 2t = \frac{9t^2}{20} - \frac{2t}{3}$$

Angular velocity at the end of 5 sec.

$$= \frac{9}{20} \times 5 \times 5 - \frac{2}{3} \times 5 = \frac{225}{20} - \frac{10}{3}$$

$$= 7.92 \text{rad/s}$$

Angular acceleration $= \frac{d}{dt}\left[\frac{9t^2}{20} - \frac{2t}{3}\right] = \frac{9}{20} \times 2t - \frac{2}{3}$

At the end of 5 sec. Angular acceleration

$$= 4.5 - 0.67 = 3.83 \text{ rad/s}^2$$

75. A wheel of mass 10kg and radius 20cm is rotating at an angular speed of 100rev/min when the motor is turned off. Neglecting the friction at the axle, calculate the force that must be applied tangentially to the wheel to bring it to rest in 10 revolutions.

Ans. : $\omega = 100\mathrm{rev/min} = \frac{5}{8}\mathrm{rev/s} = \frac{10\pi}{3}\mathrm{rad/s}$

$\theta = 10\mathrm{rev} = 20\pi\mathrm{rad},\ \mathrm{r} = 0.2\mathrm{m}$

After 10 revolutions the wheel will come to rest by a tangential force.

Therefore the angular deacceleration produced by the force $= \alpha = \frac{\omega^2}{2\theta}$

Therefore the torque by which the wheel will come to an rest $= \mathrm{I_{cm}} \times \alpha$

$\Rightarrow \mathrm{F} \times \mathrm{r} = \mathrm{I_{cm}} \times \alpha$

$\Rightarrow \mathrm{F} \times 0.2 = \frac{1}{2}\mathrm{mr}^2 \times \left[\frac{\left(\frac{10\pi}{3}\right)^2}{(2\times 20\pi)}\right]$

$\Rightarrow \mathrm{F} = \frac{1}{2} \times 10 \times 0.2 \times \frac{100\pi^2}{(9\times 2\times 20\pi)}$

$= \frac{5\pi}{18} = \frac{15.7}{18} = 0.87\mathrm{N}$

76. Two blocks of masses 10kg and 30kg are placed along a vertical line. The first block is raised through a height of 7cm. By what distance should the second mass be moved to raise the centre of mass by 1cm?

Ans. : Two masses m_1 & m_2 are kept in a vertical line.

m_1 = 10kg, m_2 = 30kg

The first block is raised through a height of 7cm.

The centre of mass is raised by 1cm.

$\therefore 1 = \frac{\mathrm{m_1y_1+m_2y_2}}{\mathrm{m_1+m_2}} = \frac{10\times 7+30\mathrm{y_2}}{40}$

$\Rightarrow 1 = \frac{70+30\mathrm{y_2}}{40}$

$\Rightarrow 70 + 30\mathrm{y_2} = 40$

$\Rightarrow 30\mathrm{y_2} = -30$

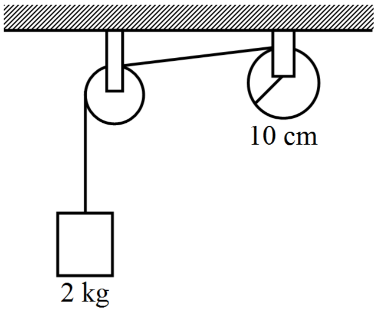
$\Rightarrow \mathrm{y_2} = -1$

The 30kg body should be displaced 1cm downward inorder to raise the centre of mass through 1cm.

77. If ice on poles melts, then what is the change in duration of day?

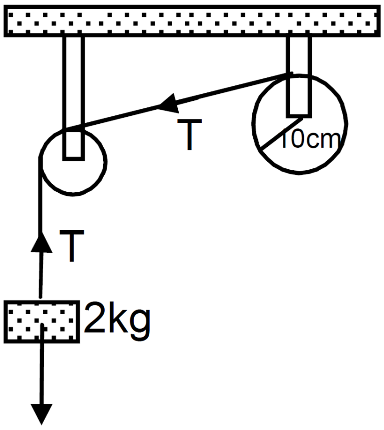
Ans. : Molten ice from poles goes into ocean and so mass is going away from axis of rotation. So, moment of inertia of earth increases and to conserve angular momentum, angular velocity (ω) decreases. So, time period of rotation increases $\left(\mathrm{T} = \frac{2\pi}{\omega}\right)$ So, net effect of global warming is increasing in the duration of day.

78. A string is wrapped on a wheel of moment of inertia 0.20kg-m^2 and radius 10cm and goes through a light pulley to support a block of mass 2.0kg as shown in figure. Find the



acceleration of the block.

Ans. :



I = 0.20kg-m^2 (Bigger pulley)

r = 10cm = 0.1m, smaller pulley is light

mass of the block, m = 2kg

therefore $\mathrm{mg} - \mathrm{T} = \mathrm{ma} \ \ldots (1)$

$\Rightarrow \mathrm{T} = \frac{\mathrm{Ia}}{\mathrm{r}^2}$

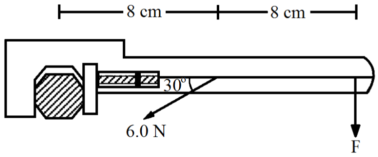
$\Rightarrow \mathrm{mg} = \left(\mathrm{m} + \frac{\mathrm{I}}{\mathrm{r}^2}\right)\mathrm{a}$

$\Rightarrow \frac{(2 \times 9.8)}{\left[2 + \left(\frac{0.2}{0.01}\right)\right]} = \mathrm{a}$

$= \frac{19.6}{22} = 0.89\mathrm{m/s}^2$

Therefore, acceleration of the block = 0.89m/s^2.

79. When a force of 6.0N is exerted at 30° to a wrench at a distance of 8cm from the nut, it is just able to loosen the nut. What force F would be sufficient to loosen it if it acts perpendicularly to the wrench at 16cm from the nut?



Ans. : A force of 6N acting at an angle of 30° is just able to loosen the wrench at a distance 8cm from it.

Therefore total torque acting at A about the point O

$= 6 \sin 30^\circ \times \left(\frac{8}{100}\right)$

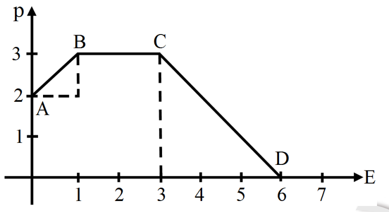
Therefore total torque required at B about the point O

$= \mathrm{F} \times \frac{16}{100}$

$\Rightarrow \mathrm{F} \times \frac{16}{100} = 6\sin 30^{\circ} \times \frac{8}{100}$

$\Rightarrow \mathrm{F} = \frac{(8\times3)}{16} = 1.5\mathrm{N}$

80. Figure shows momentum versus time graph for a particle moving along x-axis. In which region, force on the particle is large. Why?



Ans. : Net force is given by $\mathrm{F}_{\text{net}} = \frac{\mathrm{dp}}{\mathrm{dt}}$

Also, rate of change of momentum = slope of graph.

As from graph, slope AB = slope CD

And slope (BC) = slope (DE) = 0

So, force acting on the particle is equal in regions AB and CD and in regions BC and DE (which is zero).

81. Two blocks of mares 10kg and 20kg are placed on the X-axis. The first mass is moved on the axis by a distance of 2cm. By what distance should the second mass be moved to keep the position of the centre of mass unchanged?

Ans. : Two masses m_1 & m_2 are placed on the X-axis

m_1 = 10kg, m_2 = 20kg.

The first mass is displaced by a distance of 2cm.

$\therefore \overline{\mathrm{X}}_{\mathrm{cm}} = \frac{\mathrm{m_1x_1+m_2x_2}}{\mathrm{m_1+m_2}} = \frac{10\times2+20\mathrm{x_2}}{30}$

$\Rightarrow 0 = \frac{20+20\mathrm{x_2}}{30}$

$\Rightarrow 20 + 20\mathrm{x}_2 = 0$

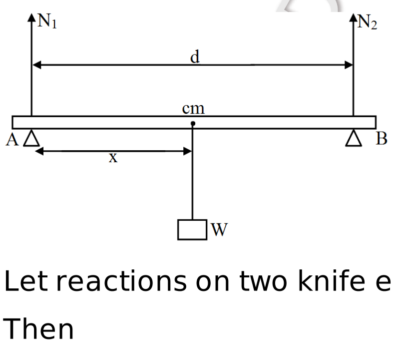
$\Rightarrow 20 = -20\mathrm{x}_2$

$\Rightarrow \mathrm{x}_2 = -1$

$\therefore$ The 2^{nd} mass should be displaced by a distance 1cm towards left so as to kept the position of centre of mass unchanged.

82. A rod of weight W is supported by two parallel knife edges A and B and is in equilibrium in a horizontal position. The distance between the knife edges is d and the centre of mass of the rod is at a distance x from A. Find the value of normal reactions at the knife edges A and B.

Ans. : The situation is shown in Fig



Let reactions on two knife edges be N_1 and N_2 respectively.

Then

$N_1 + N_2 = W$ ...(1)

and from principle of moments, taking moments at point A, we have

$N_2 \times d = W \times x$ ...(2)

Equation (2) leads

$$N_2 = \frac{W_x}{d}$$

and substituting this value in (i), we get $N_1 = W - N_2 = W - \frac{Wx}{d}$

$$= W\left(1 - \frac{x}{d}\right).$$

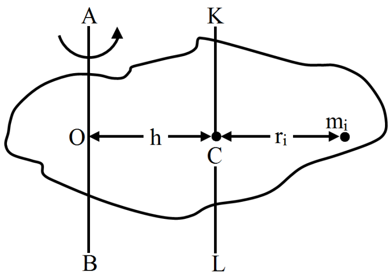
83.

a. State the theorem of parallel axis. Using it derive an expression to find the moment of inertia of a rod of mass M, length 1 about an axis perpendicular to it passing through one of its ends.

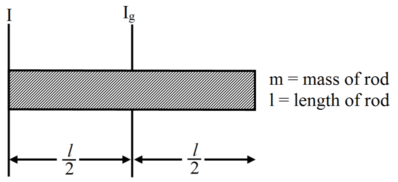
b. Find the centre of mass of a uniform L shaped lamina (a thin flat plate) with dimension as shown in fig. The mass of lamina is 3kg.

Ans. :

a. **Parallel axis theorem:** According to this theorem, the moment of inertia of a rigid body about an axis AB is equal to the sum of moments of inertia of the body about another axis KL passing through the centre of mass C of the body in a direction parallel to AB and the product of total mass M of the body and square of the perpendicular distance between the two parallel axes.



Let the mass of the rod m,



According to the theorem of parrallel axis

$$I = I_g + mL^2$$

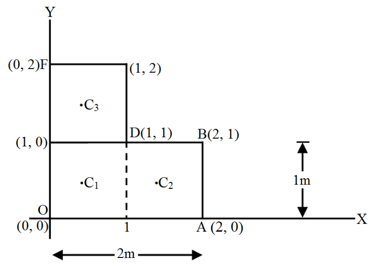
$$\because I_g = \frac{ml^2}{12}$$

$$\text{and } L = \frac{l}{2}$$

$$\Rightarrow I = \frac{ml^2}{12} + m\left(\frac{l}{2}\right)^2$$

$$I = \frac{ml^2}{12} + \frac{ml^2}{4} = \frac{ml^2}{3}$$

b. Choosing the X and Y axes as shown in the figure, the coordinates of the vertices of the L-shaped lamina are given in the figure.



The L-shaped lamina consists of three squares each of side 1m and mass 1kg ($\because$ the lamina is uniform). By symmetry the centres of mass C_1, C_2, C_3 of the squares are their geometric centres and have coordiantes $C_1\left(\frac{1}{2}, \frac{1}{2}\right), C_2\left(\frac{3}{2}, \frac{1}{2}\right)$ and $C_3\left(\frac{1}{2}, \frac{3}{2}\right)$ taking the masses of the square of be concentrated at these points, the coordinates of the centres of mass are caculated as

$$x = \frac{m-1x_1+m_2x_2+m_3x_3}{m_1+m_2+m_3}$$

$$= \frac{1\times\frac{1}{2}+1\times\frac{3}{2}+1\times\frac{1}{2}}{1+1+} = \frac{5}{6}m$$

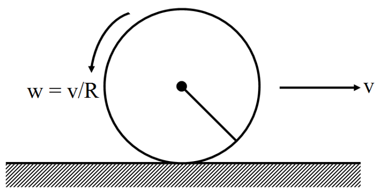
$$y = \frac{m_1y_1+m_2y_2+m_3y_3}{m_1+m_2+m_3}$$

$$= \frac{1\times\frac{1}{2}+1\times\frac{1}{2}+1\times\frac{3}{2}}{1+1+1} = \frac{5}{6}m.$$

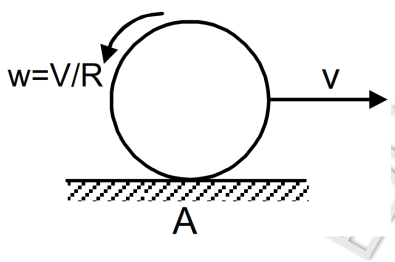
84. A solid sphere is set into motion on a rough horizontal surface with a linear speed v in the forward direction and an angular speed $\frac{v}{R}$ in the anticlockwise direction as shown in figure. Find the linear speed of the sphere:

a. When it stops rotating.

b. When slipping finally ceases and pure rolling starts.



Ans. :



a. If we take moment at A then external torque will be zero.

Therefore, the initial angular momentum = the angular momentum after rotation stops (i.e. only leniar velocity exits)

$$Mv \times R - \ell\omega = Mv_0 \times R$$

$$\Rightarrow MvR - \frac{2}{5} \times \frac{MR^2V}{R} = Mv_0R$$

$$\Rightarrow v_0 = \frac{3V}{5}$$

b. Again, after some time pure rolling starts

Therefore,

$$\Rightarrow \mathrm{M} \times \mathrm{v}_0 \times \mathrm{R} = \left(\frac{2}{5}\right)\mathrm{MR}^2 \times \left(\frac{\mathrm{V}'}{\mathrm{R}}\right) + \mathrm{Mv}'\mathrm{R}$$

$$\Rightarrow \mathrm{m} \times \left(\frac{3\mathrm{V}}{5}\right) \times \mathrm{R} = \left(\frac{2}{5}\right)\mathrm{Mv}'\mathrm{R} + \mathrm{Mv}'\mathrm{R}$$

$$\Rightarrow \mathrm{V}' = \frac{3\mathrm{V}}{7}$$

85. What is the difference between rotational kinetic energy and rolling kinetic energy? Show that rolling kinetic energy of a rolling body is given by $\frac{1}{2}\mathrm{mv}^2\left(\frac{\mathrm{K}^2}{\mathrm{r}^2}+1\right)$ where r is radius of the body and K is the radius of gyration of the body.

Ans. : Rotational K.E. is only due to the rotational motion, while K.E. under rolling is the sum of the rotational and translational kinetic energies.

K.E. in Rolling = Rotational K.E. + Translational K.E.

$= \frac{1}{2}\ \mathrm{I}\ \omega^2 + \frac{1}{2}\mathrm{mv}^2$

$= \frac{1}{2}\mathrm{mK}^2\omega^2 + \frac{1}{2}\mathrm{mv}^2$

$= \frac{1}{2}\mathrm{m}\frac{\mathrm{K}^2}{\mathrm{r}^2}\mathrm{v}^2 + \frac{1}{2}\mathrm{mv}^2$

K.E. in rolling $= \frac{1}{2}\mathrm{mv}^2\left(1+\frac{\mathrm{K}^2}{\mathrm{r}^2}\right)$

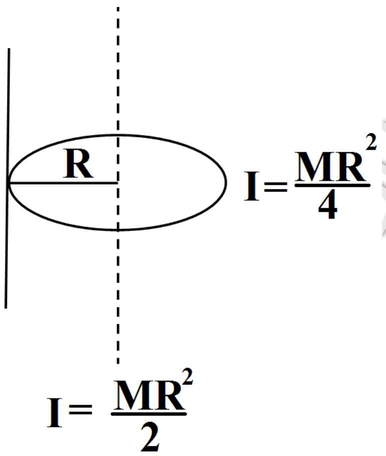
86. Given the moment of inertia of a disc of mass M and radius R about any of its diameters to be $\frac{\mathrm{MR}^2}{4}$, find its moment of inertia about an axis normal to the disc and passing through a point on its edge.

Ans. : The moment of inertia of a disc about its diameter $= \frac{\mathrm{MR}^2}{4}$

According to the theorem of perpendicular axis, the moment of inertia of a planar body (lamina) about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes concurrent with perpendicular axis and lying in the plane of the body.

The M.I. of the disc about its centre $= \frac{\mathrm{MR}^2}{4} + \frac{\mathrm{MR}^2}{4} = \frac{\mathrm{MR}^2}{2}$

The situation is shown in the given figure:



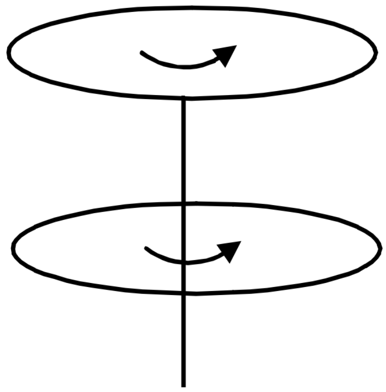
Applying the theorem of parallel axes,

The moment of inertia about an axis normal to the disc and passing through a point on its edge

$= \frac{\mathrm{MR}^2}{2} + \mathrm{MR}^2 = \frac{3\mathrm{MR}^2}{2}$

87. A wheel of moment of inertia 0.10kg-m^2 is rotating about a shaft at an angular speed of 160rev/ minute. A second wheel is set into rotation at 300rev/ minute and is coupled to the same shaft so that both the wheels finally rotate with a common angular speed of 200rev/ minute. Find the moment of inertia of the second wheel.

Ans. :



Wheel (1) has

$l_1 = 0.10\text{kg-m}^2$

$\omega_1 = 160\text{rev}/\text{min}$

Wheel (2) has

$l_2 = ?$

$\omega_2 = 300\text{rev}/\text{min}$

Given that after they are coupled, $\omega = 200\text{rev}/\text{min}$

Therefore if we take the two wheels to bean isolated system

Total external torque = 0

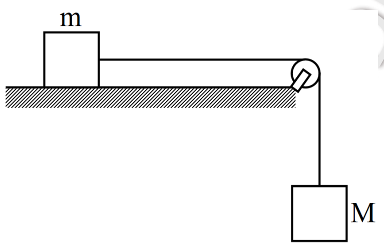
Therefore, $l_1\omega_1 + l_2\omega_2 = (l_1 + l_2)\omega$

$\Rightarrow 0.10 \times 160 + l_2 \times 300 = (0.10 + l_2) \times 200$

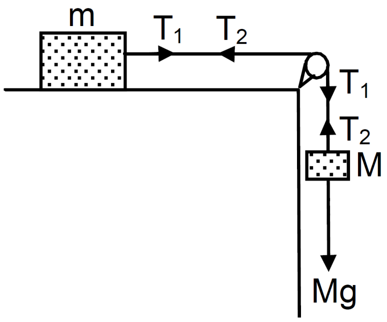
$\Rightarrow 5l_2 = 1 - 0.8$

$\Rightarrow l_2 = 0.04\text{Kg-m}^2$

88. Figure shows two blocks of masses m and M connected by a string passing over a pulley. The horizontal table over which the mass m slides is smooth. The pulley has a radius r and moment of inertia I about its axis and it can freely rotate about this axis. Find the acceleration of the mass M assuming that the string does not slip on the pulley.



Ans. : According to the question



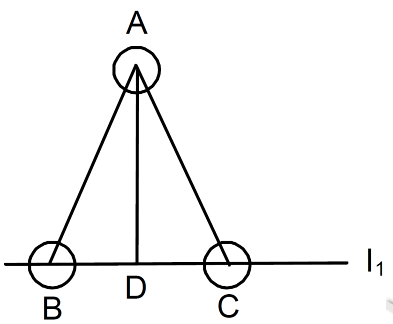
$$\mathrm{Mg} - \mathrm{T_1} = \mathrm{ma}\ \ldots(1)$$
$$\mathrm{T_2} = \mathrm{ma}\ \ldots(2)$$
$$\left(\mathrm{T_1} - \mathrm{T_2}\right) = \frac{1\mathrm{a}}{\mathrm{r}^2}\ \ldots(3)\ [\text{ because } \mathrm{a} = \mathrm{r}\alpha]\ \ldots\ \left[\mathrm{T.r} = 1\left(\frac{\mathrm{a}}{\mathrm{r}}\right)\right]$$

If we add the equation 1 and 2 we will get

$$\mathrm{Mg} + (\mathrm{T_2} - \mathrm{T_1}) = \mathrm{ma} + \mathrm{ma}\ \ldots(4)$$
$$\Rightarrow \mathrm{Mg} - \frac{\mathrm{la}}{\mathrm{r}^2} = \mathrm{Ma} + \mathrm{ma}$$
$$\Rightarrow \left(\mathrm{M} + \mathrm{m} + \frac{1}{\mathrm{r}^2}\right)\mathrm{a} = \mathrm{Mg}$$
$$\Rightarrow \mathrm{a} = \frac{\mathrm{Mg}}{\left(\mathrm{M+m+}\frac{1}{\mathrm{r}^2}\right)}$$

89. Three particles, each of mass 200g, are kept at the corners of an equilateral triangle of side 10cm. Find the moment of inertia of the system about an axis:
  a. Joining two of the particles.
  b. Passing through one of the particles and perpendicular to the plane of the particles.

Ans. :



a. Therefore, the $\perp$ distance from the axis $(\mathrm{AD}) = \frac{\sqrt{3}}{2} \times 10 = 5\sqrt{3}\mathrm{cm.}$

Therefore moment of inertia about the axis BC will be

$$\mathrm{I} = \mathrm{mr}^2 = 200\mathrm{k}\left(5\sqrt{3}\right)^2 = 200 \times 25 \times 3$$
$$= 15000\mathrm{gm} - \mathrm{cm}^2 = 1.5 \times 10^{-3}\mathrm{kg} - \mathrm{m}^2$$

b. The axis of rotation let pass through A and $\perp$ to the plane of triangle

Therefore the torque will be produced by mass B and C

Therefore net moment of inertia = I = mr^2 + mr^2

= 2 × 200 × 10^2 = 40000gm-cm^2

= 4 × 10^{-3}kg-m^2.

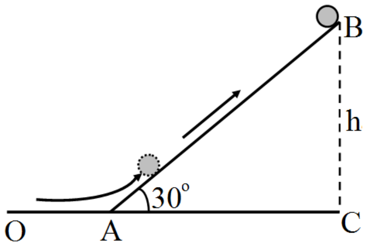
90. A solid cylinder rolls up an inclined plane of angle of inclination 30°. At the bottom of the inclined plane the centre of mass of the cylinder has a speed of 5m/s.

a. How far will the cylinder go up the plane?

b. How long will it take to return to the bottom?

Ans. :

a. A solid cylinder rolling up an inclination is shown in the following figure:



Initial velocity of the solid cylinder, v = 5m/s

Angle of inclination, $\theta = 30^{\circ}$

Height reached by the cylinder = h Energy of the cylinder at point A

$\mathrm{KE_{root}} = \mathrm{KE_{trans}}$

$\frac{1}{2}\mathrm{I}\omega^2 = \frac{1}{2}\mathrm{mv}^2$

Energy of the cylinder at point B = mgh

Using the law of conservation of energy, we can write,

$\frac{1}{2}\mathrm{I}\omega^2 = \frac{1}{2}\mathrm{mv}^2 = \mathrm{mgh}$

Moment of inertia of the solid cylinder, $\mathrm{I} = \frac{1}{2}\mathrm{mr}^2$

$\therefore\ \frac{1}{2}\left(\frac{1}{2}\mathrm{mr}^2\right)\omega^2 + \frac{1}{2}\mathrm{mv}^2 = \mathrm{mgh}$

$\frac{1}{4}\mathrm{mr}^2\omega^2 + \frac{1}{2}\mathrm{mv}^2 = \mathrm{mgh}$

But we have the relation, $\mathrm{v} = \mathrm{r}\omega$

$\therefore\ \frac{1}{4}\mathrm{v}^2 + \frac{1}{2}\mathrm{v}^2 = \mathrm{gh}$

$\frac{3}{4}\mathrm{v}^2 = \mathrm{gh}$

$\therefore\ \mathrm{h} = \frac{3}{4}\frac{\mathrm{v}^2}{\mathrm{g}}$

$= \frac{3}{4} \times \frac{5\times5}{9.8} = 1.91\mathrm{m}$

b. In $\Delta\mathrm{ABC}$

$\sin\theta = \frac{\mathrm{BC}}{\mathrm{AB}}$

$\sin 30^{\circ} = \frac{\mathrm{h}}{\mathrm{AB}}$

$\mathrm{AB} = \frac{1.91}{0.5} = 3.82\mathrm{m}$

Hence, the cylinder will travel 3.82m up the inclined plane. For radius of gyration K, the velocity of the cylinder at the instance when it rolls back to the bottom is given by the relation.

91. A ball of mass 0.50kg moving at a speed of 5.0m/s collides with another ball of mass 1.0kg. After the collision the balls stick together and remain motionless. What was the velocity of the 1.0kg block before the collision?

Ans. : Mass of the ball = m_1 = 0.5kg, Velocity of the ball = 5m/s

Mass of the another ball m_2 = 1kg

Let it's velocity = v' m/s

Using law of conservation of momentum,

$0.5 \times 5 + 1 \times v' = 0$

$\Rightarrow v' = -2.5$

$\therefore$ Velocity of second ball is 2.5m/s opposite to the direction of motion of 1st ball.

92. Two blocks of masses 10kg and 30kg are placed along a vertical line. The first block is raised through a height of 7cm. By what distance should the second mass be moved to raise the centre of mass by 1cm?

Ans. : Two masses m_1 & m_2 are kept in a vertical line.

m_1 = 10kg, m_2 = 30kg

The first block is raised through a height of 7cm.

The centre of mass is raised by 1cm.

$\therefore 1 = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{10 \times 7 + 30 y_2}{40}$

$\Rightarrow 1 = \frac{70 + 30 y_2}{40}$

$\Rightarrow 70 + 30 y_2 = 40$

$\Rightarrow 30 y_2 = -30$

$\Rightarrow y_2 = -1$

The 30kg body should be displaced 1cm downward inorder to raise the centre of mass through 1cm.

93. A 60kg man skating with a speed of 10m/s collides with a 40kg skater at rest and they cling to each other. Find the loss of kinetic energy during the collision.

Ans. : Mass of the man = m_1 = 60kg

Speed of the man = v_1 = 10m/s

Mass of the skater = m_2 = 40kg

let its velocity = v'

$\therefore 60 \times 10 + 0 = 100 \times v'$

$\Rightarrow v'$ = 6m/s

loss in $\Delta KE = \frac{1}{2} m_1 v_1^2 - \frac{1}{2} (m_1 + m_2) v^2$

$K.E. = \left(\frac{1}{2}\right) 60 \times (10)^2 - \left(\frac{1}{2}\right) \times 100 \times 36 = 1200J$

94. Find the ratio of the linear momenta of two particles of masses 1.0kg and 4.0kg if their kinetic energies are equal.

Ans. : Let the mass of the to particles be m_1 & m_2 respectively

m_1 = 1kg, m_2 = 4kg

$\therefore$ According to question

$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_2 v_2^2$

$\Rightarrow \frac{m_1}{m_2} = \frac{v_2^2}{v_1^2}$

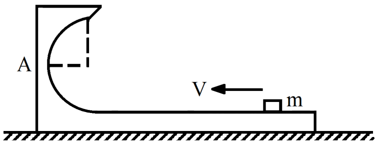
$\Rightarrow \frac{v_2}{v_1} = \sqrt{\frac{m_1}{m_2}}$

$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{m_2}{m_1}}$

Now, $\frac{m_1 v_1}{m_2 v_2} = \frac{m_1}{m_2} \times \sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{m_1}{m_2}} = \frac{\sqrt{1}}{\sqrt{4}} = \frac{1}{2}$

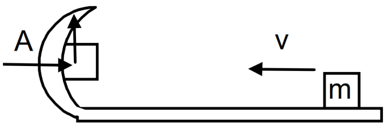
$\Rightarrow \frac{m_1 v_1}{m_2 v_2} = 1:2$

95. Figure shows a small block of mass m which is started with a speed v on the horizontal part of the bigger block of mass M placed on a horizontal floor. The curved part of the surface shown is semicircular. All the surfaces are frictionless. Find the speed of the bigger block when the smaller block reaches the point A of the surface.



Ans. : A small block of mass m which is started with a velocity V on the horizontal part of the bigger block of mass M placed on a horizontal floor.

Since the small body of mass m is started with a velocity V in the horizontal direction, so the total initial momentum at the initial position in the horizontal direction will remain same as the total final momentum at the point A on the bigger block in the horizontal direction.

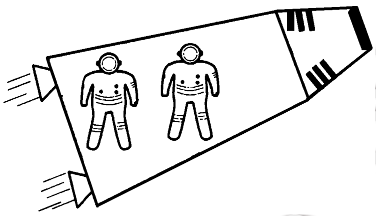


From L.C.K. m:

$$mv + M \times O = (m + M)v$$

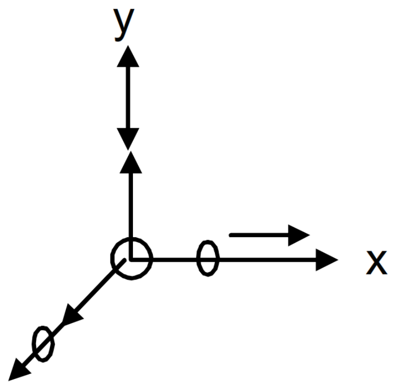
$\Rightarrow v' = \frac{mv}{M+m}$

96. A block at rest explodes into three equal parts. Two parts start moving along X and Y axes respectively with equal speeds of 10m/s. Find the initial velocity of the third part.



Ans. : As the block is exploded only due to its internal energy. So net external force during this process is 0. So the centre mass will not change. Let the body while exploded was at the origin of the co-ordinate system. If the two bodies of equal mass is moving at a speed of 10m/s in +x & +y axis direction respectively,

$\sqrt{10^2 + 10^2 + 210.10\cos 90^\circ} = 10\sqrt{2}\text{m/s}$ 45° w.r.t. + x axis



If the centre mass is at rest, then the third mass which have equal mass with other two, will move in the opposite direction (i.e. 135° w.r.t. +x- axis) of the resultant at the same velocity.

97. Two blocks of mares 10kg and 20kg are placed on the X-axis. The first mass is moved on the axis by a distance of 2cm. By what distance should the second mass be moved to keep the position of the centre of mass unchanged?

Ans. : Two masses m_1 & m_2 are placed on the X-axis

m_1 = 10kg, m_2 = 20kg.

The first mass is displaced by a distance of 2cm.

$\therefore \overline{X}_{cm} = \frac{m_1x_1+m_2x_2}{m_1+m_2} = \frac{10\times2+20x_2}{30}$

$\Rightarrow 0 = \frac{20+20x_2}{30}$

$\Rightarrow 20 + 20x_2 = 0$

$\Rightarrow 20 = -20x_2$

$\Rightarrow x_2 = -1$

$\therefore$ The 2^{nd} mass should be displaced by a distance 1cm towards left so as to kept the position of centre of mass unchanged.

98. In a typical Indian Bugghi (a luxury cart drawn by horses), a wooden plate is fixed on the rear on which one person can sit. A bugghi of mass 200kg is moving at a speed of 10km/h. As it overtakes a school boy walking at a speed of 4km/h, the boy sits on the wooden plate. If the mass of the boy is 25kg, what will be the new velocity of the bugghi?

Ans. : Mass of the boggli = 200kg, V_B = 10km/ hour.

Mass of the boy = 2.5kg & V_{Boy} = 4km/ hour.

If we take the boy & boggle as a system then total momentum before the process of sitting will remain constant after the process of sitting.

$m_b V_b = m_{boy}V_{boy} = (m_b + m_{boy})v$

$\Rightarrow 200 \times 10 + 25 \times 4 = (200 + 25) \times v$

$\Rightarrow v = \frac{2100}{225} = \frac{28}{3} = 9.3\text{m/sec}$

99. During a heavy rain, hailstones of average size 1.0cm in diameter fall with an average speed of 20m/s. Suppose 2000 hailstones strike every square meter of a 10m × 10m roof perpendicularly in one second and assume that the hailstones do not rebound. Calculate the average force exerted by the falling hailstones on the roof. Density of a hailstone is $900 kg/m^3$.

Ans. : $\mathrm{d} = 1\mathrm{cm},\ \mathrm{v} = 20\mathrm{cm},\ \mathrm{u} = 0,\ \rho = 900\mathrm{kg/m}^3 = 0.9\mathrm{gm/cm}^3$

Volume $= \left(\frac{4}{3}\right)\pi \mathrm{r}^3 = \left(\frac{4}{3}\right)\pi(0.5)^3 = 0.5238\mathrm{cm}^3$

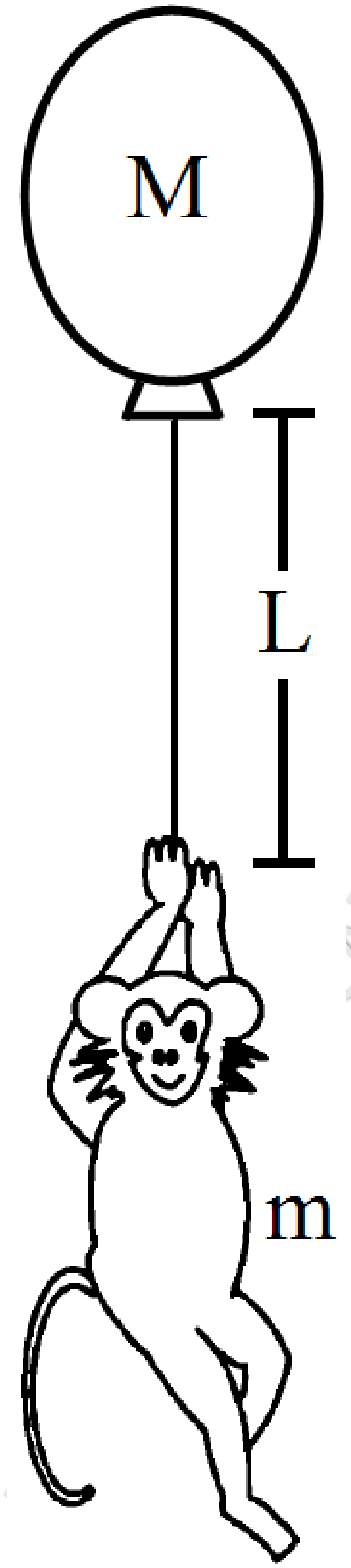
$\therefore$ Mass $= \mathrm{v}_\rho = 0.5238 \times 0.9 = 0.4714258\mathrm{gm}$

$\therefore$ Mass of 2000 hailstone = 2000 × 0.4714 = 947.857

$\therefore$ Rate of change in momentum per unit area = 947.857 × 2000 = 19N/m^3

$\therefore$ Total force exerted = 19 × 100 = 1900N

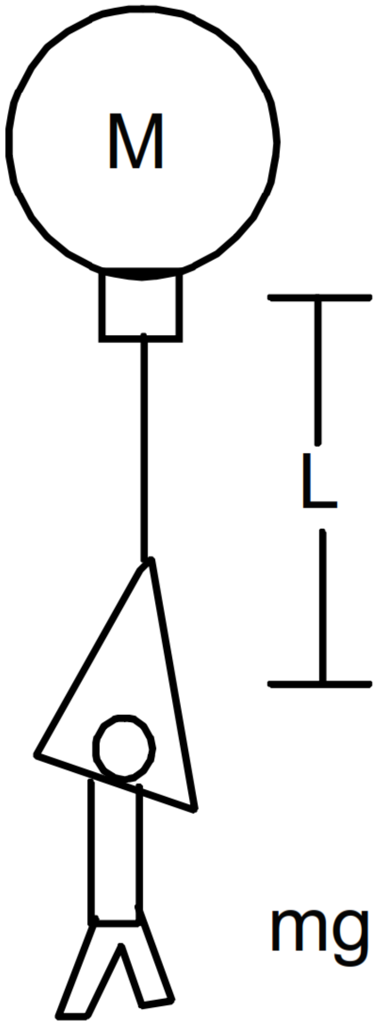
100. The balloon, the light rope and the monkey shown in figure are at rest in the air. If the monkey reaches the top of the rope, by what distance does the balloon descend ? Mass of the balloon = M, mass of the monkey = m and the length of the rope ascended by



the monkey = L.

Ans. : Initially the monkey & balloon are at rest.

So the CM is at ‘P’



When the monkey descends through a distance 'L'

The CM will shift

$t_0 = \frac{m \times L + M \times 0}{M+m} = \frac{mL}{M+m}$ from P

So, the balloon descends through a distance $\frac{mL}{M+m}$

101. A ball falls on the ground from a height of 2.0m and rebounds up to a height of 1.5m. Find the coefficient of restitution.

Ans. : Let the velocity of the ball falling from height h_1 be u (when it approaches the ground).

Velocity on the ground $u = \sqrt{2gh_1}$

Let the velocity of ball when it separates from the ground be v. (Assuming it goes up to height h_2)

$\Rightarrow v = \sqrt{2gh_2}$

$= \sqrt{2 \times 9.8 \times 1.5}$

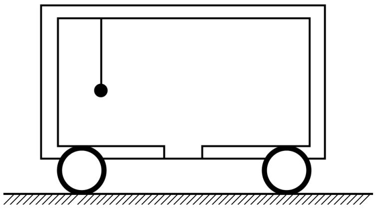
Let the coefficient of restituti be e.

We khow, v = eu

$\Rightarrow e = \frac{\sqrt{2 \times 9.8 \times 1.5}}{\sqrt{2 \times 9.8 \times 2}} = \frac{\sqrt{3}}{2}$

Hence, the coefficient of restitution is $\frac{\sqrt{3}}{2}$

102. A cart of mass M is at rest on a frictionless horizontal surface and a pendulum bob of mass m hangs from the roof of the cart. The string breaks, the bob falls on the floor, makes several collisions on the floor and finally lands up in a small slot made in the floor. The horizontal distance between the string and the slot is L. Find the displacement of the cart during this process.



Ans. : Let the bob fall at A, The mass of bob = m.

The mass of cart = M.

Initially their centre of mass will be at

$\frac{\mathrm{m \times L + M \times 0}}{\mathrm{M+m}} = \left(\frac{\mathrm{m}}{\mathrm{M+m}}\right)\mathrm{L}$

Distance from P,

When, the bob falls in the slot the CM is at a distance 'O' from P.

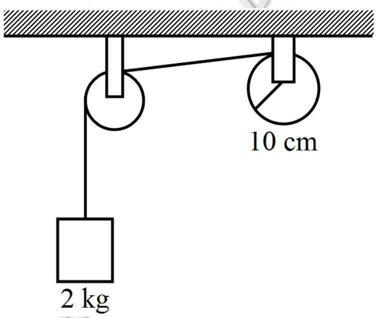
Shift in $\mathrm{CM} = 0 - \frac{\mathrm{mL}}{\mathrm{M+m}} = -\frac{\mathrm{mL}}{\mathrm{M+m}}$ towards left

$= \frac{\mathrm{mL}}{\mathrm{M+m}}$ towards right.

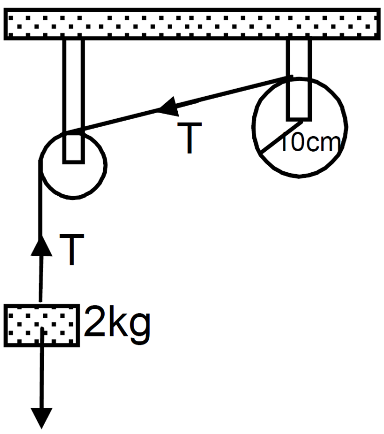
But there is no external force in horizontal direction.

So the cart displaces a distance $\frac{\mathrm{mL}}{\mathrm{M+m}}$ towards right.

103. A string is wrapped on a wheel of moment of inertia 0.20kg-m^2 and radius 10cm and goes through a light pulley to support a block of mass 2.0kg as shown in figure. Find the acceleration of the block.



Ans. :



I = 0.20kg-m^2 (Bigger pulley)

r = 10cm = 0.1m, smaller pulley is light mass of the block, m = 2kg

therefore $\mathrm{mg} - \mathrm{T} = \mathrm{ma} \ \ldots (1)$

$\Rightarrow \mathrm{T} = \frac{\mathrm{la}}{\mathrm{r}^2}$

$\Rightarrow \mathrm{mg} = \left(\mathrm{m} + \frac{\mathrm{l}}{\mathrm{r}^2}\right)\mathrm{a}$

$\Rightarrow \frac{(2\times 9.8)}{\left[2+\left(\frac{0.2}{0.01}\right)\right]} = \mathrm{a}$

$= \frac{19.6}{22} = 0.89\mathrm{m/s}^2$

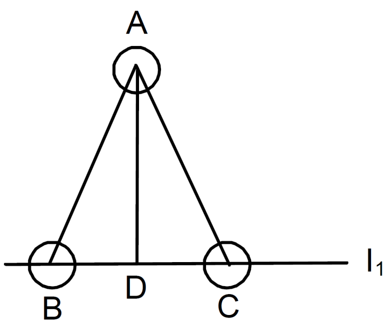
Therefore, acceleration of the block = $0.89\mathrm{m/s}^2$.

104. Three particles, each of mass 200g, are kept at the corners of an equilateral triangle of side 10cm. Find the moment of inertia of the system about an axis:

a. Joining two of the particles.

b. Passing through one of the particles and perpendicular to the plane of the particles.

Ans. :



a. Therefore, the $\perp$ distance from the axis $(\mathrm{AD}) = \frac{\sqrt{3}}{2} \times 10 = 5\sqrt{3}\mathrm{cm}.$

Therefore moment of inertia about the axis BC will be

$\mathrm{I} = \mathrm{mr}^2 = 200\mathrm{k}\left(5\sqrt{3}\right)^2 = 200 \times 25 \times 3$

$= 15000\mathrm{gm} - \mathrm{cm}^2 = 1.5 \times 10^{-3}\mathrm{kg} - \mathrm{m}^2$

b. The axis of rotation let pass through A and $\perp$ to the plane of triangle

Therefore the torque will be produced by mass B and C

Therefore net moment of inertia = I = $\mathrm{mr}^2 + \mathrm{mr}^2$

$= 2 \times 200 \times 10^2 = 40000\mathrm{gm\text{-}cm}^2$

$= 4 \times 10^{-3}\mathrm{kg\text{-}m}^2$.

*** Given Section consists of questions of 5 marks each. [185]**

105. A 3m long ladder weighing 20 kg leans on a frictionless wall. Its feet rest on the floor 1 m from the wall as shown in Fig.6.27. Find the reaction forces of the wall and the floor.

Ans. :

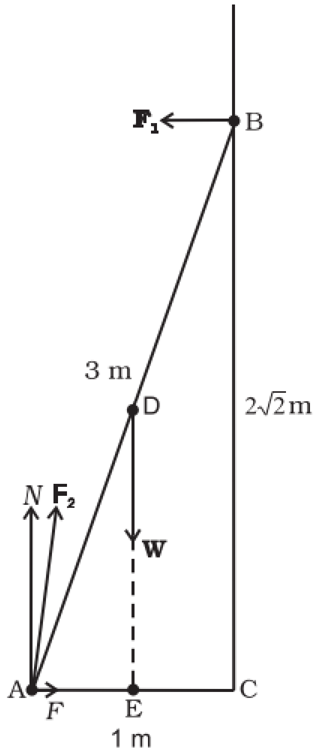


Fig. 6.27

The ladder AB is $3m$ long, its foot A is at distance $AC = 1m$ from the wall. From Pythagoras theorem, $BC = 2\sqrt{2}m$. The forces on the ladder are its weight W acting at its centre of gravity D, reaction forces F_1 and F_2 of the wall and the floor respectively. Force F_1 is perpendicular to the wall, since the wall is frictionless. Force F_2 is resolved into two components, the normal reaction N and the force of friction F. Note that F prevents the ladder from sliding away from the wall and is therefore directed toward the wall.

For translational equilibrium, taking the forces in the vertical direction,

$N - W = 0 \qquad \text{(i)}$

Taking the forces in the horizontal direction,

$F - F_1 = 0 \qquad \text{(ii)}$

For rotational equilibrium, taking the moments of the forces about A,

$2\sqrt{2}F_1 - (1/2)W = 0 \qquad \text{(iii)}$

Now $\quad W = 20g = 20 \quad 9.8N = 196.0N$

From (i) $N = 196.0N$

From (iii) $F_1 = W/4\sqrt{2} = 196.0/4\sqrt{2} = 34.6N$

From (ii) $F = F_1 = 34.6N$

$F_2 = \sqrt{F^2 + N^2} = 199.0N$

The force F_2 makes an angle α with the horizontal,
$\tan\alpha = N/F = 4\sqrt{2}, \quad \alpha = \tan^{-1}(4\sqrt{2}) \approx 80^\circ$

106. From a uniform disk of radius R, a circular hole of radius $\frac{R}{2}$ is cut out. The centre of the hole is at $\frac{R}{2}$ from the centre of the original disc. Locate the centre of gravity of the resulting flat body.

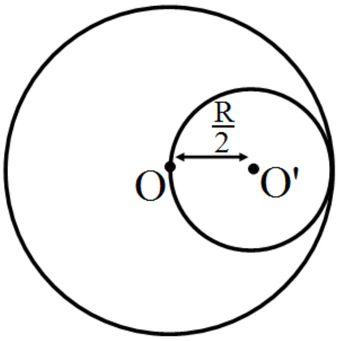
Ans. : $\frac{R}{6}$, from the original centre of the body and opposite to the centre of the cut portion.

Mass per unit area of the original disc $= \sigma$

Radius of the original disc = R

Mass of the original disc, $M = \pi R^2 \sigma$

The disc with the cut portion is shown in the following figure:



Radius of the smaller disc $= \frac{R}{2}$

Mass of the smaller disc, $M' = \pi\left(\frac{R}{2}\right)^2 \sigma = \frac{\pi R^2 \sigma}{4} = \frac{M}{4}$

Let O and O′ be the respective centres of the original disc and the disc cut off from the original. As per the definition of the centre of mass, the centre of mass of the original disc is supposed to be concentrated at O, while that of the smaller disc is supposed to be concentrated at O′. It is given that,

$OO' = \frac{R}{2}$

After the smaller disc has been cut from the original, the remaining portion is considered to be a system of two masses. The two masses are,

M (concentrated at O), and $-M' = \left(\frac{M}{4}\right)$ concentrated at O′ (The negative sign indicates that this portion has been removed from the original disc).

Let x be the distance through which the centre of mass of the remaining portion shifts from point O.

The relation between the centres of masses of two masses is given as,

$x = \frac{(m_1 r_1 + m_2 r_2)}{(m_1 + m_2)}$

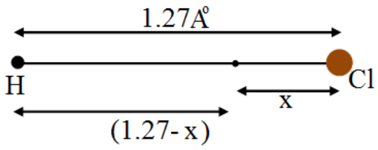
For the given system, we can write,

$x = \frac{\left[M \times 0 - M' \times \left(\frac{R}{2}\right)\right]}{(M + M'))} = \frac{-R}{6}$

(The negative sign indicates that the centre of mass gets shifted toward the left of point O).

107. In the HCl molecule, the separation between the nuclei of the two atoms is about $1.27\mathring{A}\left(1\mathring{A}=10^{-10}\mathrm{m}\right)$. Find the approximate location of the CM of the molecule, given that a chlorine atom is about 35.5 times as massive as a hydrogen atom and nearly all the mass of an atom is concentrated in its nucleus.

Ans. :



Distance between H and Cl atoms $= 1.27\mathring{A}$

Mass of H atom = m

Mass of Cl atom = 35.5m

Let the centre of mass of the system lie at a distance x from the Cl atom.

Distance of the centre of mass from the H atom = (1.27 - x)

Let us assume that the centre of mass of the given molecule lies at the origin. Therefore, we can have,

[m(1.27 - x) + 35.5mx] / (m + 35.5m) = 0

m(1.27 - x) + 35.5mx = 0

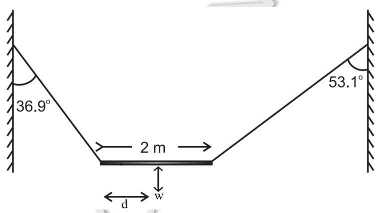
1.27 - x = -35.5x

$\therefore\ \mathrm{x} = -1.27/(35.5-1) = -0.37\mathring{A}$

Here, the negative sign indicates that the centre of mass lies at the left of the molecule.

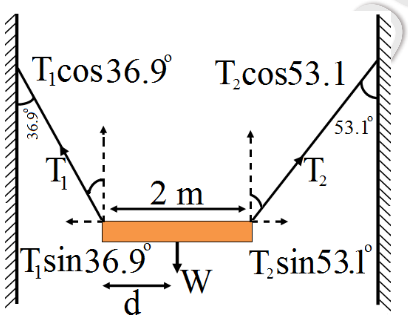
Hence, the centre of mass of the HCl molecule lies $0.37\mathring{A}$ from the Cl atom.

108. A non-uniform bar of weight W is suspended at rest by two strings of negligible weight as shown in. The angles made by the strings with the vertical are 36.9° and 53.1° respectively. The bar is 2m long. Calculate the distance d of the centre of gravity of the



bar from its left end.

Ans. : The free body diagram of the bar is shown in the following figure:



Length of the bar, l = 2m

T_1 and T_2 are the tensions produced in the left and right strings respectively. At translational equilibrium, we have,

$$\mathrm{T_1}\sin 36.9^\circ = \mathrm{T_2}\sin 53.1$$

$\frac{T_1}{T_2} = \frac{\sin 53.1^\circ}{\sin 36.9}$

$= \frac{0.800}{0.600} = \frac{4}{3}$

$T_1 = \frac{4}{3} T_2$

For rotational equilibrium, on taking the torque about the centre of gravity, we have,

$T_1 \cos 36.9 \times d = T_2 \cos 53.1(2 - d)$

$T_1 \times 0.800d = T_2 0.600(2 - d)$

$\frac{4}{3} \times T_2 \times 0.800d = T_2 \left[0.600 \times 2 - 0.600d\right]$

$1.067d + 0.6d = 1.2$

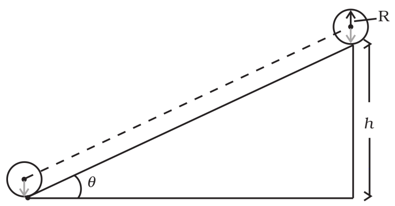
$\therefore\ d = \frac{1.2}{1.67}$

$= 0.72m$

Hence, the C.G. (centre of gravity) of the given bar lies 0.72m from its left end.

109. Prove the result that the velocity v of translation of a rolling body (like a ring, disc, cylinder or sphere) at the bottom of an inclined plane of a height h is given by $v^2 = \frac{2gh}{\left(\frac{1+k^2}{R^2}\right)}$ using dynamical consideration (i.e. by consideration of forces and torques). **Note:** k is the radius of gyration of the body about its symmetry axis, and R is the radius of the body. The body starts from rest at the top of the plane.

Ans. : A body rolling on an inclined plane of height h,is shown in the following figure:



m = Mass of the body.

R = Radius of the body.

K = Radius of gyration of the body.

v = Translational velocity of the body.

h =Height of the inclined plane.

g = Acceleration due to gravity.

Total energy at the top of the plane, E_1= mgh

Total energy at the bottom of the plane, E_b = KE_{rot} + KE_{trans}

$= \frac{1/2}{\omega^2} + \frac{1}{2}mv^2$

But I = mk^2 and $\omega = \frac{v}{R}$

$\therefore\ E_b = \frac{1mk^2v^2}{2R^2} + \frac{1}{2}mv^2$

From the law of conservation of energy, we have,

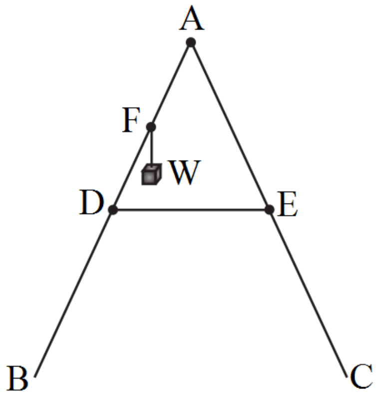
$E_T = E_b$

$mgh = \left(\frac{1mv^2 1+k^2}{2R^2}\right)$

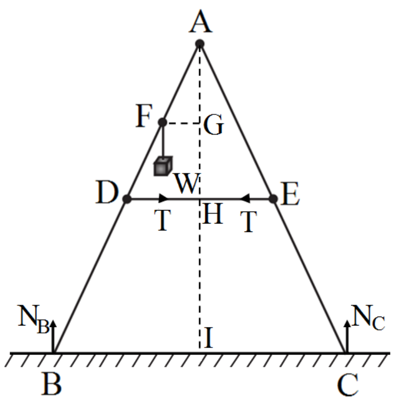
$$\therefore\ v^2 = \frac{2gh}{\left(\frac{1+k^2}{R^2}\right)}$$

Hence, the given result is proved.

110. As shown in the two sides of a step ladder BA and CA are 1.6m long and hinged at A. A rope DE, 0.5m is tied half way up. A weight 40kg is suspended from a point F, 1.2m from B along the ladder BA. Assuming the floor to be frictionless and neglecting the weight of the ladder, find the tension in the rope and forces exerted by the floor on the ladder. (Take g = 9.8m/s^2) (**Hint:** Consider the equilibrium of each side of the ladder separately).



Ans. : The given situation can be shown as:



N_B = Force exerted on the ladder by the floor point B

N_C = Force exerted on the ladder by the floor point C

T = Tension in the rope

BA = CA = 1.6m

DE = 0. 5m

BF = 1.2m

Mass of the weight, m = 40kg

Draw a perpendicular from A on the floor BC. This intersects DE at mid-point H.

ΔABI and ΔAIC are similar

$\therefore$ BI = IC

Hence, I is the mid-point of BC.

DE || BC

BC = 2 × DE = 1m

AF = BA - BF = 0.4m ....(i)

D is the mid-point of AB.

Hence, we can write,

$$\mathrm{AD} = \left(\frac{1}{2}\right) \times \mathrm{BA} = 0.8\mathrm{m} \ldots \mathrm{(ii)}$$

Using equations (i) and (ii), we get,

FE = 0.4m

Hence, F is the mid-point of AD.

FG || DH and F is the mid-point of AD. Hence, G will also be the mid-point of AH.

$\Delta\mathrm{AFG}$ and $\Delta\mathrm{ADH}$ are similar

$$\therefore \ \frac{\mathrm{FG}}{\mathrm{DH}} = \frac{\mathrm{AF}}{\mathrm{AD}}$$

$$\frac{\mathrm{FG}}{\mathrm{DH}} = \frac{0.4}{0.8} = \frac{1}{2}$$

$$= \left(\frac{1}{2}\right) \times 0.25 = 0.125\mathrm{m}$$

In $\Delta\mathrm{ADH},$

AH = $(AD^2 - DH^2)^{1/2}$

= $(0.8^2 - 0.25^2)^{1/2}$ = 0.76m

For translational equilibrium of the ladder, the upward force should be equal to the downward force.

$N_C + N_B$ = mg = 392 .....(iii)

For rotational equilibrium of the ladder, the net moment about A is,

$-N_B \times BI + mg \times FG + N_C \times CI + T \times AG - T \times AG = 0$

$-N_B \times 0.5 + 40 \times 9.8 \times 0.125 + N_C \times 0.5 = 0$

$(N_C - N_B) \times 0.5 = 49$

$N_C - N_B$ = 98 ....(iv)

Adding equations (iii) and (iv), we get,

N_C = 245N

N_B = 147N

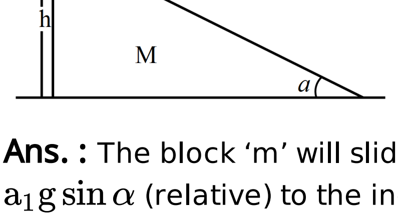
For rotational equilibrium of the side AB, consider the moment about A.

$-N_B \times BI + mg \times FG + T \times AG = 0$

$-245 \times 0.5 + 40 \times 9.8 \times 0.125 + T \times 0.76 = 0$

T = 96.7N

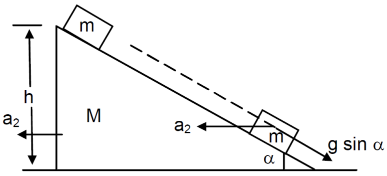
111. A block of mass m is placed on a triangular block of mass M, which in turn is placed on a horizontal surface as shown in figure. Assuming frictionless surfaces find the velocity of the triangular block when the smaller block reaches the bottom end.



Ans. : The block ‘m’ will slide down the inclined plane of mass M with acceleration $\mathrm{a_1 g \sin\alpha}$ (relative) to the inclined plane.

The horizontal component of a_1 will be, $\mathrm{a_x = g\sin\alpha\cos\alpha}$, for which the block M will accelerate towards left. Let, the acceleration be a_2.

According to the concept of centre of mass, (in the horizontal direction external force is zero).



$ma_x = (M + m)a_2$

$\Rightarrow a_2 = \frac{ma_x}{M+m} = \frac{mg\sin\alpha\cos\alpha}{M+m} \quad \ldots(1)$

So, the absolute (Resultant) acceleration of 'm' on the block 'M' along the direction of the incline will be,

$a = g\sin\alpha - a_2\cos\alpha$

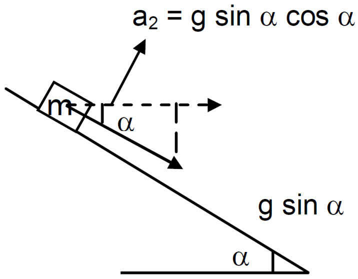
$= g\sin\alpha - \frac{mg\sin\alpha\cos^2\alpha}{M+m}$

$= g\sin\alpha\left[1 - \frac{m\cos^2\alpha}{M+m}\right]$

$= g\sin\alpha\left[\frac{M-m-m\cos^2\alpha}{M+m}\right]$

So, $a = g\sin\alpha\left[\frac{M+m\sin^2\alpha}{M+m}\right] \quad \ldots(2)$

Let, the time taken by the block 'm' to reach the bottom end be 't'.



Now, $S = ut + \left(\frac{1}{2}\right)at^2$

$\Rightarrow \frac{h}{\sin\alpha} = \left(\frac{1}{2}\right)at^2$

$\Rightarrow t = \sqrt{\frac{2}{a\sin\alpha}}$

So, the velocity of the bigger block after time 't' will be.

$V_m = u + a_2 t$

$= \frac{mg\sin\alpha\cos\alpha}{M+m}\sqrt{\frac{2}{a\sin\alpha}}$

$= \sqrt{\frac{2m^2g^2h\sin^2\alpha\cos^2\alpha}{(M+m)^2 a\sin\alpha}}$

Now, subtracting the value of a from equation (2) we get,

$$V_m = \left[\frac{2m^2g^2h\sin^2\alpha\cos^2\alpha}{(M+m)^2 a\sin\alpha} \times \frac{(M+m)}{g\sin\alpha(M+m\sin^2\alpha)}\right]^{\frac{1}{2}}$$

or $V_m = \left[\frac{2m^2g^2h\cos^2\alpha}{(M+m)(M+m\sin^2\alpha)}\right]^{\frac{1}{2}}$

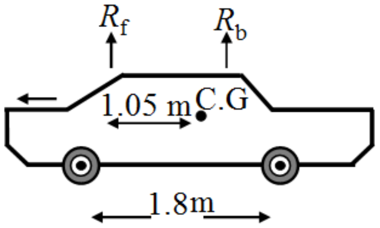
112. A car weighs 1800kg. The distance between its front and back axles is 1.8m. Its centre of gravity is 1.05m behind the front axle. Determine the force exerted by the level ground on each front wheel and each back wheel.

Ans. : Mass of the car, m = 1800kg

Distance between the front and back axles, d = 1.8m

Distance between the C.G. (centre of gravity) and the back axle = 1.05m

The various forces acting on the car are shown in the following figure:



R_f and R_b are the forces exerted by the level ground on the front and back wheels respectively.

At translational equilibrium:

$R_f + R_b = mg$

= 1800 × 9.8

= 17640N ....(i)

For rotational equilibrium, on taking the torque about the C.G.,

We have,

$R_f(1.05) = R_b(1.8 - 1.05)$

$\frac{R_b}{R_f} = \frac{7}{5}$

$R_b = 1.4\ R_f$ ....(ii)

Solving equations (i) and (ii), we get

$1.4R_f + R_f = 17640$

$R_f = 7350N$

$\therefore R_b = 17640 - 7350 = 10290NT$

Therefore, the force exerted on each front wheel $= \frac{7350}{2} = 3675\mathrm{N}$ and,

The force exerted on each back wheel $= \frac{10290}{2} = 5145\mathrm{N}$

113. Prove the result that the velocity v of translation of a rolling body (like a ring, disc, cylinder or sphere) at the bottom of an inclined plane of height h given by $\mathrm{v}^2 = \frac{2gh}{\left(1+\frac{K^2}{R^2}\right)}$ using dynamical consideration. Note K is the radius of gyration of the body about its symmetry axis, and R is the radius of the body. The body starts from rest at the top of the plane.

Ans. : Consider a mass 'm' capable of rolling down the inclined plane from a vertical height 'h'. Rolling comprises of transitory motion of centre of mass and rotation produced by frictional force. Resolving the forces acting on the mass, we have

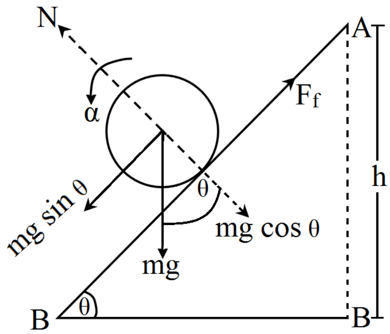
Normal reaction force $\mathrm{N} = \mathrm{mg}\cos\theta$ and

$= \mathrm{mg}\sin\theta - \mathrm{F_f} = \mathrm{ma}\ldots(1)$

Torque = F_f r and also $\tau = \mathrm{I}\alpha = \frac{l\alpha}{r}\ldots(2)$

$F_{fr} = \frac{l\alpha}{r}$

$\Rightarrow F_f = \frac{Ia}{r} \ldots (2)$ where, r is the radius of the body



Using F_f in (i), we have

$$mg \sin \theta = \left(m + \frac{l}{r^2}\right)a$$

$$a = \frac{mg \sin \theta}{m\left(1+\frac{K^2}{r^2}\right)}$$

$$= \frac{g \sin \theta}{\left(1+\frac{K^2}{r^2}\right)}[\therefore I = mK^2]$$

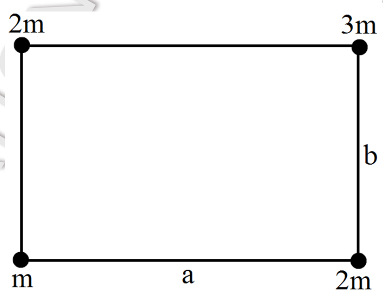
Where K is the radius of gyration of the body about its symmetry axis

using a, in $v^2 = u^2 + 2as$, we have

$$v = \sqrt{\frac{2g \sin \theta l}{\left(1+\frac{K^2}{r^2}\right)}}$$

$$= \sqrt{\frac{2gh}{\left(1+\frac{K^2}{r^2}\right)}}[\therefore i \sin = h][u = 0].$$

114. Four bodies have been arranged at the corners of a rectangle shown in figure. Find the centre of mass of the system.



Ans. : Let m_1 = m, m_2 = 2m, m_3 = 3m, m_4= 2m

Let mass m, be at origin.

$$\therefore \text{ For } m_1; x_1 = 0, y_1 = 0$$

$$\text{For } m_2; x_2 = a\hat{i}$$

$$y_2 = 0$$

$$\text{For } m_4; x_4 = 0, y - 4 = b\hat{j}$$

Coordinates of COM of the system are

$$x = \frac{m_1x_1+m_2x_2+m_3x_3+m_4x_4}{m_1+m_2+m_3+m_4}$$

$$= \frac{m\times 02m\times a\hat{i}+3m\times a\hat{i}+2m\times 0}{m+2m+3m+2m}$$

$x = \frac{5ma\hat{i}}{8m} = \frac{5a\hat{i}}{8}$

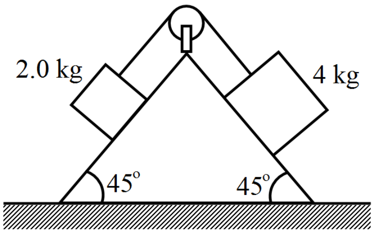
$y = \frac{m_1y_1+m_2y_2+m_3y_3+m_4y_4}{m_+m_2+m_3+m_4}$

$= \frac{m\times0+2m\times0+3m\times b\hat{j}+2m\times b\hat{j}}{m+2m+3m+2m}$

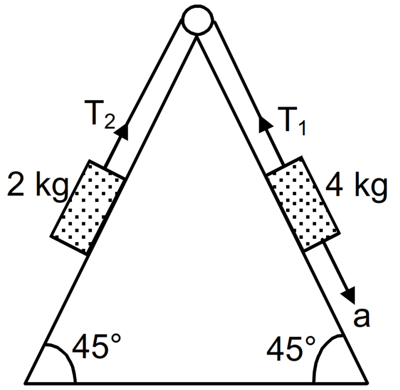
$= \frac{5mb\hat{j}}{8m} = \frac{5}{8}b\hat{j}$

$\therefore$ Centre of mass of system is $\frac{5}{8}(a\hat{i} + b\hat{j})$

115. The pulley shown in figure has a radius 10cm and moment of inertia 0.5kg-m^2 about its axis. Assuming the inclined planes to be frictionless, calculate the acceleration of the 4.0kg block.



Ans. :



$m_1 g \sin\theta - T_1 = m_1 a \ \ldots(1)$

$(T_1 - T_2) = \frac{Ia}{r^2} \ \ldots(2)$

$T_2 - m_2 g \sin\theta = m_2 a \ \ldots(3)$

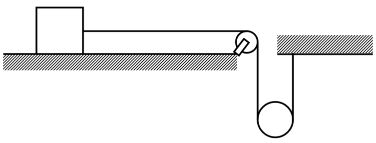
Adding the equations (1) and (3) we will get

$m_1 g \sin\theta + (T_2 - T_1) - m_2 g \sin\theta = (m_1 + m_2)a$

$\Rightarrow (m_1 - m_2) g \sin\theta = \left(m_1 + m_2 + \frac{1}{r^2}\right)a$

$\Rightarrow a = \frac{(m_1-m_2)g\sin\theta}{\left(m_1+m_2+\frac{1}{r^2}\right)} = 0.248 = 0.25ms^{-2}$

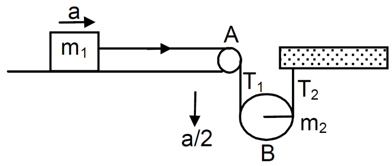
116. The descending pulley shown in figure has a radius 20cm and moment of inertia 0.20kg-m^2. The fixed pulley is light and the horizontal plane frictionless. Find the acceleration of the block if its mass is 1.0kg.



Ans. : A is light pulley and B is the descending pulley having I = 0.20kg-m^2 and r = 0.2m

Mass of the block = 1kg

According to the equation



$T_1 = m_1 a \ \ldots (1)$

$(T_2 - T_1) r = l\alpha \ \ldots (2)$

$m_2 g - \frac{m_2 a}{2} = T_1 + T_2 \ \ldots (3)$

$T_2 - T_1 = \frac{la}{2R^2} = \frac{5a}{2}$ and $T_1 = a$ $\left(\text{because } \alpha = \frac{a}{2R}\right)$

$\Rightarrow T_2 = \frac{7}{2} a$

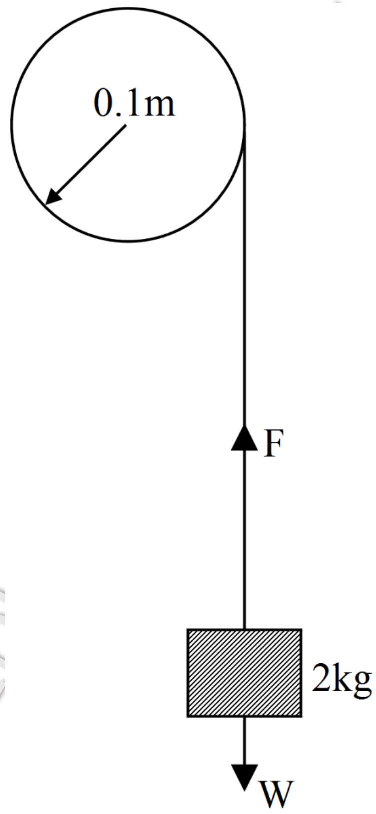
$\Rightarrow m_2 g = \frac{m_2 a}{2} + \frac{7}{2} a + a$

$\Rightarrow \frac{2l}{r^2 g} = \frac{\frac{2l}{r^2} a}{2} + \frac{9}{2} a \left(\frac{1}{2} mr^2 = l\right)$

$\Rightarrow 98 = 5a + 4.5a$

$\Rightarrow a = \frac{98}{9.5} = 10.3 ms^2$

117. The moment of inertia of a solid flywheel about its axis is $0.1 kg\text{-}m^2$. A tangential force of 2kg/ wt is applied round the circumference of the flywheel with the help of a string and mass arrangement as shown in Fig. If the radius of the wheel is 0.1m, find the acceleration of the mass.



Ans. : Let a be the linear acceleration of the mass and T the tension in the spring. It is clear that

$mg - T = ma \ldots (1)$

Let the angular acceleration of the flywheel be α. The couple applied to the flywheel is

$l\alpha = TR \ldots (2)$

The linear acceleration a and angular acceleration are related to each other as

$\mathrm{r} = \mathrm{R}\ \alpha \ldots (3)$

Combining Eq. (1), (2) and (3), we get

$\mathrm{mg} - \frac{\mathrm{l}\alpha}{\mathrm{R}} = \mathrm{m\ R}\ \alpha$

$\alpha = \frac{\mathrm{m_g R}}{(1+\mathrm{m\ R})^2} \ldots (4)$

It is given that = 2kg, R = 0.1m and I = 0.1kgm^2.Substituting these values, We get

$\alpha = \frac{2\times9.8\times0.1}{(0.1+2\times0.1^2)}\mathrm{rad\ s^{-2}}$

$= 16.7\ \mathrm{rad\ s^{-2}}$

118. A threaded rod with 12turns/ cm and diameter 1.18cm is mounted horizontally. A bar with a threaded hole to match the rod is screwed onto the rod. The bar spins at 216rev/ min. How long will it take for the bar to move 1.50cm along the rod?

Ans. : Here, distance between two consecutive threads = pitch $= \frac{1}{12}\mathrm{cm.}$

Total distance to be moved = 1.5m

$\therefore$ No. of rotations $= \frac{1.5}{\frac{1}{12}} = 18$

Total angle of turing, $\theta = 18 \times 2\pi = 36\pi$ radian

angular speed, $\omega = 2\pi\mathrm{n} = 2\pi \times \frac{216}{60} = 7.2\pi\mathrm{rad/s}$

$\therefore$ time taken, $\mathrm{t} = \frac{\theta}{\omega} = \frac{36\pi}{7.2\pi}\mathrm{5s.}$

119. A disc of radius R is cut out from a larger disc of radius 2R in such a way that the edge of the hole touches the edge of the disc. Locate the centre of mass of the residual disc.

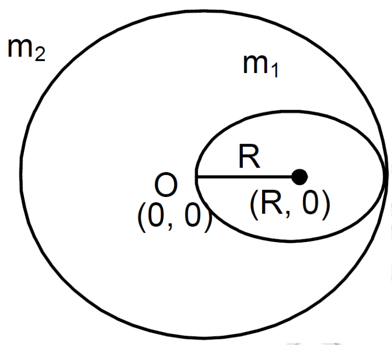
Ans. : Let '0' be the origin of the system.

R = radius of the smaller disc.

2R = radius of the bigger disc.

The smaller disc is cut out from the bigger disc.

As from the figure:



$\mathrm{m_1} = \pi\mathrm{R^2T}\rho,\ \mathrm{x_1} = \mathrm{R},\ \mathrm{y_1} = 0$

$\mathrm{m_2} = \pi(2\mathrm{R})^2\mathrm{T}\rho,\ \mathrm{x_2} = 0,\ \mathrm{y_2} = 0$

$\left(\frac{-\pi\mathrm{R^2T}\rho\mathrm{R}+0}{\pi\mathrm{R^2T}\rho+\pi(2\mathrm{R})^2\mathrm{T}\rho\mathrm{R}}, \frac{0+0}{\mathrm{m_1+m_2}}\right)$

$= \left(\frac{-\pi\mathrm{R^2T}\rho\mathrm{R}}{3\pi\mathrm{R^2T}\rho}, 0\right) = \left(-\frac{\mathrm{R}}{3}, 0\right)$

C.M. is at $\frac{\mathrm{R}}{3}$ from the centre of bigger disc away from the centre of the hole.

120. A uniform rod of mass m and length l is struck at an end by a force F perpendicular to the rod for a short time interval t. Calculate:

a. The speed of the centre of mass.
b. The angular speed of the rod about the centre of mass.
c. The kinetic energy of the rod.
d. The angular momentum of the rod about the centre of mass after the force has stopped to act. Assume that t is so small that the rod does not appreciably change its direction while the force acts.

Ans. : A uniform rod of mass m length ℓ is struck at an end by a force F. $\perp$ to the rod for a short time t:

a. Speed of the centre of mass:

$$\mathrm{mv} = \mathrm{Ft}$$

$$\Rightarrow \mathrm{v} = \frac{\mathrm{Ft}}{\mathrm{m}}$$

b. The angular speed of the rod about the centre of mass:

$$\ell\omega - \mathrm{r} \times \mathrm{p}$$

$$\Rightarrow \left(\frac{\mathrm{m}\ell}{12}\right) \times \omega = \left(\frac{1}{2}\right) \times \mathrm{mv}$$

$$\Rightarrow \left(\frac{\mathrm{m}\ell}{12}\right) \times \omega = \left(\frac{1}{2}\right) \ell\omega^2$$

$$\Rightarrow \omega = \frac{\mathrm{m}\ell}{6\mathrm{Ft}}$$

c. $\mathrm{K.E.} = \left(\frac{1}{2}\right)\mathrm{mv}^2 + \left(\frac{1}{2}\right)\ell\omega^2$

$$= \left(\frac{1}{2}\right) \times \mathrm{m}\left(\frac{\mathrm{Ft}}{\mathrm{m}}\right)^2 + \left(\frac{1}{2}\right)\ell\omega^2$$

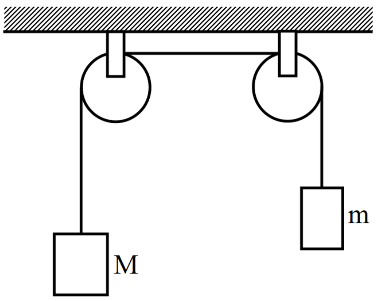
$$= \left(\frac{1}{2}\right) \times \mathrm{m} \times \left(\frac{\mathrm{F}^2\mathrm{t}^2}{\mathrm{m}^2}\right) + \left(\frac{1}{2}\right) \times \left(\frac{\mathrm{m}\ell^2}{12}\right)\left[36 \times \left(\frac{\mathrm{F}^2\mathrm{t}^2}{\mathrm{m}^2\ell^2}\right)\right]$$

$$= \frac{\mathrm{F}^2\mathrm{t}^2}{2\mathrm{m}} + \frac{3}{2}\left(\frac{\mathrm{F}^2\mathrm{t}^2}{\mathrm{m}}\right) = 2\left(\frac{\mathrm{F}^2\mathrm{t}^2}{\mathrm{m}}\right)$$

d. Angular momentum about the centre of mass:

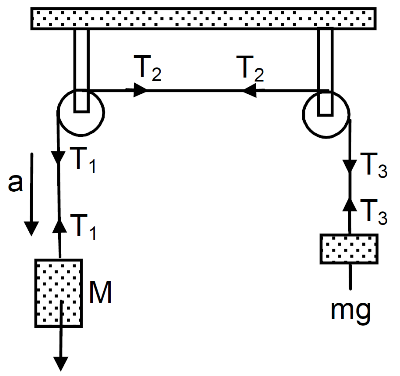
$$\mathrm{L} = \mathrm{mvr} = \mathrm{m} \times \frac{\mathrm{Ft}}{\mathrm{m}} \times \left(\frac{1}{2}\right) = \frac{\mathrm{F}\ell\mathrm{t}}{2}$$

121. The pulleys in figure are identical, each having a radius R and moment of inertia I.



Find the acceleration of the block M.

Ans. : According to the question



$$Mg - T_1 = Ma \ \ldots (1)$$
$$(T_1 - T_2)R = \frac{la}{R}$$
$$\Rightarrow (T_2 - T_1) = \frac{la}{R^2} \ \ldots (2)$$
$$(T_2 - T_3)R = \frac{la}{R^2} \ \ldots (3)$$
$$\Rightarrow T_3 - mg = ma \ \ldots (4)$$

By adding equation (2) and (3) we will get,

$$\Rightarrow (T_1 - T_3) = 2\frac{la}{R^2} \ \ldots (5)$$

By adding equation (1) and (4) we will get,

$$-mg + Mg + (T_3 - T_1) = Ma + ma \ \ldots (6)$$

Substituting the value for T_3 - T_1 we will get

$$\Rightarrow Mg - mg = Ma + ma + \frac{2la}{R^2}$$
$$\Rightarrow a = \frac{(M-m)G}{\left(M+m+\frac{2l}{R^2}\right)}$$

122. A comet revolves around the sun in a highly elliptical orbit having a minimum distance of 7×10^{10}m and a maximum distance of 1.4×10^{13}m. If its speed while nearest to the Sun is 60km s^{-1}, find its linear speed when situated farthest from the Sun.

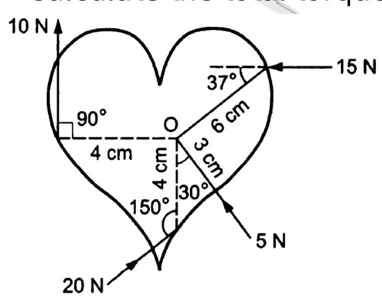
Ans. : Let mass of comet be M and its angular speed be w when situated at a distance r from the Sun, then its angular momentum L = I w = Mr^2w

If v be the linear speed, then L = Mr^2w = Mrv

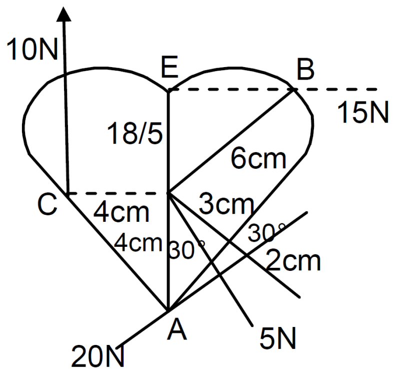
In accordance with conservation law of angular momentum, we can write that

$$Mr_1v_1 = mr_2v_2$$
$$\therefore v_2 = \frac{r_1v_1}{r_2} = \frac{7\times10^{10}m\times60km/s}{1.4\times10^{13}m}$$
$$= 0.3km/s \text{ or } 300m/s.$$

123. Calculate the total torque acting on the body shown in figure about the point O.



Ans. : Torque about a point = Total force × perpendicular distance from the point to that force.



Let anticlockwise torque = +ve

And clockwise acting torque = -ve

Force acting at the point B is 15N

Therefore torque at O due to this force

$= 15 \times 6 \times 10^{-2} \times \sin 37^{\circ}$

$= 15 \times 6 \times 10^{-2} \times \frac{3}{5} = 0.54\text{N-m}$ (anticlock wise)

Force acting at the point C is 10N

Therefore, torque at O due to this force

= $10 \times 4 \times 10^{-2}$ = 0.4N-m (clockwise)

Force acting at the point A is 20N

Therefore, Torque at O due to this force $= 20 \times 4 \times 10^{-2} \times \sin 30^{\circ}$

$= 20 \times 4 \times 10^{-2} \times \frac{1}{2} = 0.4\text{N-m}$ (anticlockwise)

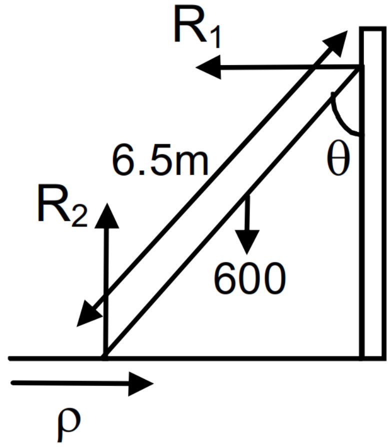
Therefore resultant torque acting at 'O' = 0.54 - 0.4 + 0.4 = 0.54N-m.

124. A 6.5m long ladder rests against a vertical wall reaching a height of 6.0m. A 60kg man stands half way up the ladder.

a. Find the torque of the force exerted by the man on the ladder about the upper end of the ladder.

b. Assuming the weight of the ladder to be negligible as compared to the man and assuming the wall to be smooth, find the force exerted by the ground on the ladder.

Ans. :



m = 60kg, ladder length = 6.5m, height of the wall = 6m

Therefore torque due to the weight of the body

a. $\tau = \frac{600 \times 6.5}{2 \sin \theta} = \mathrm{i}$

$\Rightarrow \tau = \frac{600 \times 6.5}{2 \times \sqrt{\left[1 - \left(\frac{6}{6.5}\right)^2\right]}}$

$\Rightarrow \tau = 735\mathrm{N\text{-}m}$

b. $\mathrm{R}_2 = \mathrm{mg} = 60 \times 9.8$

$\mathrm{R}_1 = \mu \mathrm{R}_2$

$\Rightarrow 6.5\mathrm{R}_1 \cos \theta = 60\mathrm{g} \sin \theta \times \frac{6.5}{2}$

$\Rightarrow \mathrm{R}_1 = 60\mathrm{g} \tan \theta = 60\mathrm{g} \times \left(\frac{2.5}{12}\right)$ [Because $\tan \theta = \frac{2.5}{6}$]

$\Rightarrow \mathrm{R}_1 = \left(\frac{25}{2}\right)\mathrm{g} = 122.5\mathrm{N}$

125. A grindstone has a moment of inertia of 6kg m^2. A constant torque is applied and the grindstone is found to have a speed of 150rpm, 10s. after starting from rest. Calculate the torque.

Ans. : Here, Moment of inertia of grindstone, I = 6kgm^2

Initial angular velocity $\omega_1 = 0$

Final angular velocity,

$\omega_2 = 2\pi \mathrm{n} = 2\pi \times \frac{150}{60}$

$= 5\pi \text{ rad/sec}^2$

Time for which torque acts,

$\mathrm{t} = 10\mathrm{sec.}$

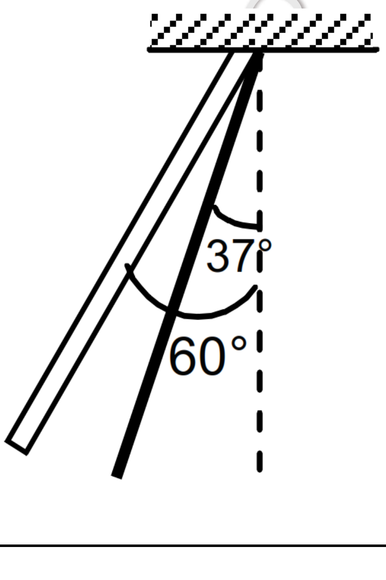
$\therefore$ Angular acceleration $(\alpha) = \frac{\omega_2 - \omega_1}{\mathrm{t}}$

$= \frac{5\pi - 0}{10} = \frac{\pi}{2} \text{rad/sec}^2$

As $\tau = \mathrm{I}\alpha$

$\therefore \tau = 6 \times \frac{\pi}{2} = 3\pi \mathrm{Ns.}$

126. A uniform rod pivoted at its upper end hangs vertically. It is displaced through an angle of 60° and then released. Find the magnitude of the force acting on a particle of mass dm at the tip of the rod when the rod makes an angle of 37° with the vertical.



Ans. :

Let l = length of the rod, and m = mass of the rod.

Applying energy principle

$\left(\frac{1}{2}\right)l\omega^2 - 0 = mg\left(\frac{1}{2}\right)(\cos 37^\circ - \cos 60^\circ)$

$\Rightarrow \frac{1}{2} \times \frac{ml^2}{3}\omega^2$

$= mg \times \frac{1}{2}\left(\frac{4}{5} - \frac{1}{2}\right)t$

$\Rightarrow \omega^2 = \frac{9g}{10l} = 0.9\left(\frac{g}{l}\right)$

Again $\left(\frac{ml^2}{3}\right)\alpha = mg\left(\frac{1}{2}\right)\sin 37^\circ = mgl \times \frac{3}{5}$

$\therefore \alpha = 0.9\left(\frac{g}{l}\right)$ = angular acceleration.

So, to find out the force on the particle at the tip of the rod

F_i = centrifugal force $= (dm)\omega^2 l = 0.9(dm)g$

F_t = tangential force $= (dm)\alpha l = 0.9(dm)g$

So, total force $F = \sqrt{\left(F_i^2 + F_t^2\right)} = 0.9\sqrt{2}(dm)g$

127. A bullet of mass 25g is fired horizontally into a ballistic pendulum of mass 5.0kg and gets embedded in it. If the centre of the pendulum rises by a distance of 10cm, find the speed of the bullet.

Ans. : Mass of bullet = 25g = 0.025kg.

Mass of pendulum = 5kg.

The vertical displacement h = 10cm = 0.1m

Let it strike the pendulum with a velocity u.

Let the final velocity be v.

$\Rightarrow mu = (M + m)v.$

$\Rightarrow v = \frac{m}{(M+m)}u$

$= \frac{0.025}{5.025} \times u = \frac{u}{201}$

Using conservation of energy.

$0 - \left(\frac{1}{2}\right)(M + m).\, v^2 = -(M + m)g \times h$

$\Rightarrow \frac{u^2}{(201)^2} = 2 \times 10 \times 0.1 = 2$

$\Rightarrow u = 201 \times \sqrt{2} = 280 m/sec$

128. A disc of radius R is cut out from a larger disc of radius 2R in such a way that the edge of the hole touches the edge of the disc. Locate the centre of mass of the residual disc.

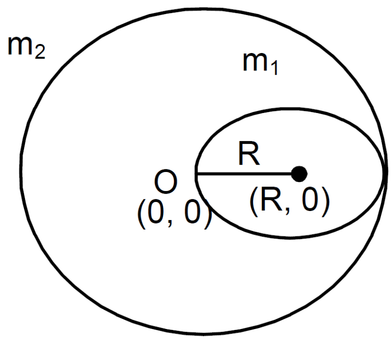
Ans. : Let '0' be the origin of the system.

R = radius of the smaller disc.

2R = radius of the bigger disc.

The smaller disc is cut out from the bigger disc.

As from the figure:



$m_1 = \pi R^2 T\rho,\ x_1 = R,\ y_1 = 0$

$m_2 = \pi(2R)^2 T\rho,\ x_2 = 0,\ y_2 = 0$

$\left(\frac{-\pi R^2 T\rho R + 0}{\pi R^2 T\rho + \pi(2R)^2 T\rho R}, \frac{0+0}{m_1+m_2}\right)$

$= \left(\frac{-\pi R^2 T\rho R}{3\pi R^2 T\rho}, 0\right) = \left(-\frac{R}{3}, 0\right)$

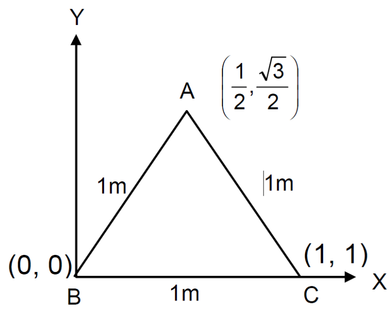
C.M. is at $\frac{R}{3}$ from the centre of bigger disc away from the centre of the hole.

129. Three particles of masses 1.0kg, 2.0kg and 3.0kg are placed at the corners A, B and C respectively of an equilateral triangle ABC of edge 1m. Locate the centre of mass of the system.

Ans. : $m_1 = 1\text{kg},\ m_2 = 2\text{kg},\ m_3 = 3\text{kg}$

$x_1 = 0,\ x_2 = 1,\ x_3 = \frac{1}{2}$

$y_1 = 0,\ y_2 = 0,\ y_3 = \frac{\sqrt{3}}{2}$



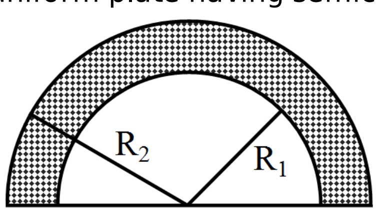
The position of centre of mass is,

$\text{CM} = \left(\frac{m_1x_1+m_2x_2+m_3x_3}{m_1+m_2+m_3}, \frac{m_1y_1+m_2y_2+m_3y_3}{m_1+m_2+m_3}\right)$

$= \left(\frac{(1\times0)+(2\times1)\left(3\times\frac{1}{2}\right)}{1+2+3}, \frac{(1\times0)+(2\times0)+\left(3\times\frac{\sqrt{3}}{2}\right)}{1+2+3}\right)$

$= \left(\frac{7}{12}, \frac{3\sqrt{3}}{12}\right)$ from the point B.

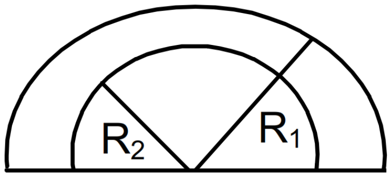
130. Find the centre of mass of a uniform plate having semicircular inner and outer boundaries of radii R_1 and R_2.



Ans. : The centre of mass of the blate will be on the symmetrical axis.

$$\Rightarrow \bar{y}_{cm} = \frac{\left(\frac{\pi R_2^2}{2}\right)\left(\frac{4R_2}{3\pi}\right) - \left(\frac{\pi R_1^2}{2}\right)\left(\frac{4R_1}{3\pi}\right)}{\frac{\pi R_2^2}{2} - \frac{\pi R_1^2}{2}}$$

$$= \frac{\left(\frac{2}{3}\right)R_2^3 - \left(\frac{2}{3}\right)R_1^3}{\frac{\pi}{2}\left(R_2^2 - R_1^2\right)} = \frac{4}{3\pi}\,\frac{(R_2 - R_1)\left(R_2^2 + R_1^2 + R_1R_2\right)}{(R_2 - R_1)(R_2 + R_1)}$$



$= \frac{4}{3\pi}\,\frac{\left(R_2^2 + R_1^2 + R_1R_2\right)}{R_1 + R_2}$ above the centre.

131. A gun is mounted on a railroad car. The mass of the car, the gun, the shells and the operator is 50m where m is the mass of one shell. If the velocity of the shell with respect to the gun (in its state before firing) is 200m/s, what is the recoil speed of the car after the second shot? Neglect friction.

Ans. : A gun is mounted on a railroad car. The mass of the car, the gun, the shells and the operator is 50m where m is the mass of one shell. The muzzle velocity of the shells is 200m/s.

Initial, $V_{cm} = 0$.

$\therefore 0 = 49m \times v + m \times 200$

$\Rightarrow v = \frac{-200}{49} m/s$

$\therefore \frac{200}{49} m/s$ towards left.

When another shell is fired, then the velocity of the car, with respect to the platform is,

$\Rightarrow v' = \frac{200}{49} m/s$ towards left.

When another shell is fired, then the velocity of the car, with respect to the platform is,

$\Rightarrow v' = \frac{200}{48} m/s$ towards left.

$\therefore$ Velocity of the car w.r.t the earth is $\left(\frac{200}{49} + \frac{200}{48}\right) m/s$ towards left.

132. A ball of mass m is dropped onto a floor from a certain height. The collision is perfectly elastic and the ball rebounds to the same height and again falls. Find the average force exerted by the ball on the floor during a long time interval.

Ans. : It is given that the mass of the ball is m.

Let the ball be dropped from a height h.

The speed of ball before the collision is v_1.

$\therefore v_1 = \sqrt{2gh}$

The speed of ball after the collision is v_2.

$v_2 = \sqrt{2gh}$

Rate of change of velocity = acceleration

$\Rightarrow a = \frac{2\sqrt{2gh}}{t}$

$\therefore$ Force, $F = \frac{m \times 2\sqrt{2gh}}{t} \ldots\ldots (1)$

Using Newton's laws of motion, We can write:

$v = \sqrt{2gh}, s = h, u = 0$

$\Rightarrow \sqrt{2gh} = gt$

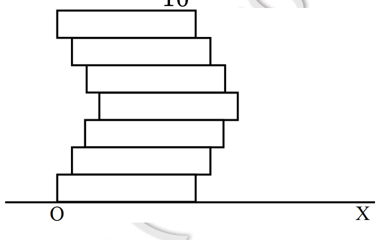
$\Rightarrow t = \sqrt{\frac{2h}{g}}$

$\therefore$ Total time $= 2\sqrt{\frac{2h}{g}}$

Substituting this value of time t in equation (1) we get:

F = mg

133. Seven homogeneous bricks, each of length L, are arranged as shown in figure. Each brick is displaced with respect to the one in contact by $\frac{L}{10}$. Find the x-coordinate of the centre of mass relative to the origin shown.

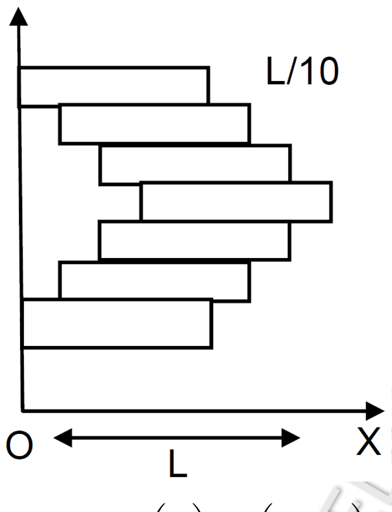


Ans. : Let 'O' (0, 0) be the origin of the system.

Each brick is mass 'M' & length 'L'.

Each brick is displaced w.r.t. one in contact by $\frac{L}{10}$

$\therefore$ The X-coordinate of the centre of mass.



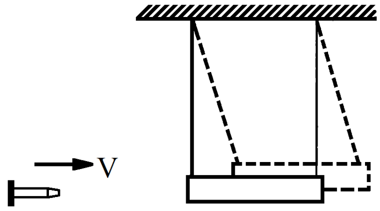
$$\overline{X}_{cm} = \frac{m\left(\frac{L}{2}\right) + m\left(\frac{L}{2} + \frac{L}{10}\right) + m\left(\frac{L}{2} + \frac{2L}{10}\right) + m\left(\frac{L}{2} + \frac{3L}{10}\right) + m\left(\frac{L}{2} + \frac{3L}{10} - \frac{L}{10}\right) + m\left(\frac{L}{2} + \frac{L}{10}\right) + M\left(\frac{L}{2}\right)}{7m}$$

$$= \frac{\frac{L}{2} + \frac{L}{2} + \frac{L}{10} + \frac{L}{2} + \frac{L}{5} + \frac{L}{2} + \frac{3L}{10} + \frac{L}{2} + \frac{L}{5} + \frac{L}{2} + \frac{L}{10} + \frac{L}{2}}{7}$$

$$= \frac{\frac{7L}{2} + \frac{5L}{10} + \frac{2L}{5}}{7}$$

$$= \frac{35L + 5L + 4L}{10 \times 7} = \frac{44L}{70} = \frac{11}{35}L$$

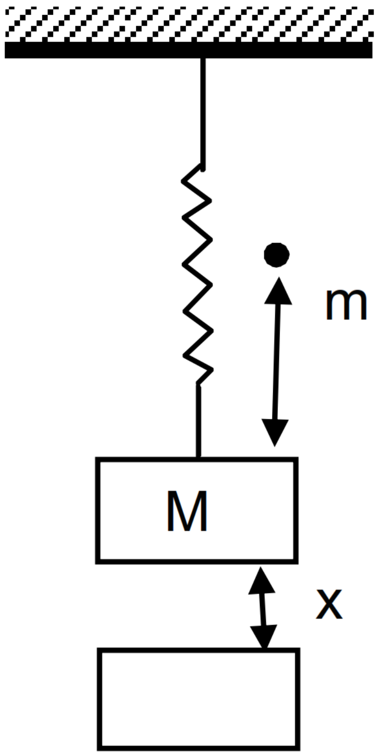
134. A block of mass 200g is suspended through a vertical spring. The spring is stretched by 1.0cm when the block is in equilibrium. A particle of mass 120g is dropped on the block from a height of 45cm. The particle sticks to the block after the impact. Find the maximum extension of the spring. Take g = 10m/s^2.



Ans. : Mass of block

Block of the particle = m = 120gm = 0.12kg.

In the equilibrium condition, the spring is stretched by a distance x = 1.00cm = 0.01m.



⇒ 0.2 × g = K. x.

⇒ 2 = K × 0.01

⇒ K = 200N/m.

The velocity with which the particle m will strike M is given by u

$$= \sqrt{2 \times 10 \times 0.45} = \sqrt{9} = 3\mathrm{m/sec}$$

So, after the collision, the velocity of the particle and the block is

$$\mathrm{V} = \frac{0.12 \times 3}{0.32} = \frac{9}{8}\mathrm{m/sec}$$

Let the spring be stretched through an extra deflection of δ.

$$0 - \left(\frac{1}{2}\right) \times 0.2 \times \left(\frac{81}{64}\right)$$

$$= 0.32 \times 10 \times \delta - \left(\frac{1}{2} \times 200 \times (\delta + 0.1)^2 - \left(\frac{1}{2}\right) \times 200 \times (0.01)^2\right)$$

Solving the above equation we get

$$\delta - 0.045 = 4.5\mathrm{cm}$$

135. A uniform disc of radius R is put over another uniform disc of radius 2R of the same thickness and density. The peripheries of the two discs touch each other. Locate the centre of mass of the system.

Ans. : Let the centre of the bigger disc be the origin.

2R = Radius of bigger disc

R = Radius of smaller disc

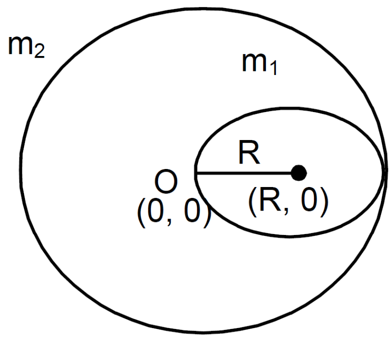
$m_1 = \pi R^2 \times T \times \rho$

$m_2 = \pi(2R)^2 \times T \times \rho$

where T = Thickness of the two discs

$\rho =$ Density of the two discs

$\therefore$ The position of the centre of mass



$\left(\frac{m_1x_1+m_2x_2}{m_1+m_2}, \frac{m_1y_1+m_2y_2}{m_1+m_2}\right)$

$x_1 = R,\ y_1 = 0$

$x_2 = 0,\ y_2 = 0$

$\left(\frac{\pi R^2 T\rho R+0}{\pi R^2 T\rho+\pi(2R)^2 T\rho}, \frac{0}{m_1+m_2}\right) = \left(\frac{\pi R^2 T\rho R}{5\pi R^2 T\rho}, 0\right) = \left(\frac{R}{5}, 0\right)$

At $\frac{R}{5}$ from the centre of bigger disc towards the centre of smaller disc.

136. A bullet of mass 20g travelling horizontally with a speed of 500m/s passes through a wooden block of mass 10.0kg initially at rest on a level surface. The bullet emerges with a speed of 100m/s and the block slides 20cm on the surface before coming to rest. Find the friction coefficient between the block and the surface.

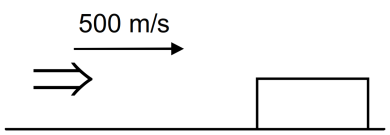
Ans. : Mass of bullet m = 0.02kg.

Initial velocity of bullet V_1 = 500m/s

Mass of block, M = 10kg.

Initial velocity of block u_2 = 0.

Final velocity of bullet = 100m/s = v.



Let the final velocity of block when the bullet emerges out, if block = v'.

$mv_1 + Mu_2 = mv + Mv'$

$\Rightarrow 0.02 \times 500 = 0.02 \times 100 + 10 \times v'$

$\Rightarrow v' = 0.8 m/s$

After moving a distance 0.2m it stops.

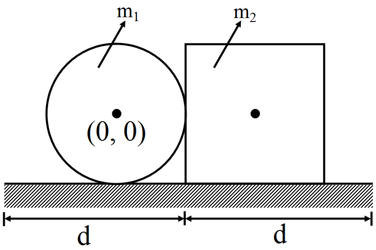
$\Rightarrow$ change in K.E. = Work done

$$\Rightarrow 0 - \left(\frac{1}{2}\right) \times 10 \times (0.8)^2$$

$$= -\mu \times 10 \times 10 \times 0.2$$

$$\Rightarrow \mu = 0.16$$

137. A square plate of edge d and a circular disc of diameter d are placed touching each other at the midpoint of an edge of the plate as shown in figure. Locate the centre of mass of the combination, assuming same mass per unit area for the two plates.



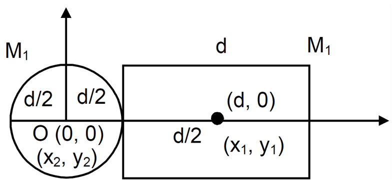
Ans. : Let m be the mass per unit area.

$\therefore$ Mass of the square plate = $M_1 = d^2 m$

Mass of the circular disc $= M_2 = \frac{\pi d^2}{4} m$

Let the centre of the circular disc be the origin of the system.

$\therefore$ Position of centre of mass



$$= \left(\frac{d^2 m d + \pi\left(\frac{d^2}{4}\right) m \times 0}{d^2 m + \pi\left(\frac{d^2}{4}\right) m}, \frac{0+0}{M_1+M_2}\right)$$

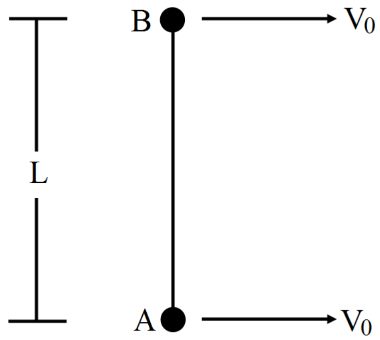
$$= \left(\frac{d^3 m}{d^2 m\left(1+\frac{\pi}{4}\right)}, 0\right)$$

$$= \left(\frac{4d}{\pi+4}, 0\right)$$

The new centre of mass is $\left(\frac{4d}{\pi+4}\right)$ right of the centre of circular disc.

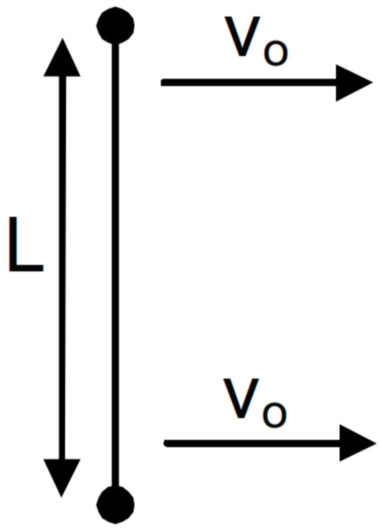
138. Two small balls A and B, each of mass m, are joined rigidly to the ends of a light rod of lengh L (figure). The system translates on a frictionless horizontal surface with a velocity vo in a direction perpendicular to the rod. A particle P of mass m kept at rest on the surface sticks to the ball A as the ball collides with it. Find:

a. The linear speeds of the balls A and B after the collision.
b. The velocity of the centre of mass C of the system A + B + P.
c. The angular speed of the system about C after the collision.



[**Hint:** The light rod will exert a force on the ball B only along its length.]

Ans. : Two balls A & B, each of mass m are joined rigidly to the ends of a light of rod of length L. The system moves in a velocity v_0 in a direction $\perp$ to the rod. A particle P of mass m kept at rest on the surface sticks to the ball A as the ball collides with it.



a. The light rod will exert a force on the ball B only along its length. So collision will not affect its velocity.

B has a velocity = v_0

If we consider the three bodies to be a system.

Applying L.C.L.M.

Therefore $mv_0 = 2mv'$

$\Rightarrow v' = \frac{v_0}{2}$

Therefore a has velocity $\frac{v_0}{2}$

b. if we consider the three bodies to be a system.

Therefore, net external force = 0

Therefore $V_{cm} = \frac{m \times v_0 + 2m\left(\frac{v_0}{2}\right)}{m+2m}$

$= \frac{mv_0 + mv_0}{3m} = \frac{2v_0}{3}$ (along the initial velocity as before collision)

c. The velocity of (A + P) w.r.t. the centre of mass $= \frac{2v_0}{3} - \frac{v_0}{2} = \frac{v_0}{6}$ &

The velocity of B w.r.t. the centre of mass $v_0 - \frac{2v_0}{3} = \frac{v_0}{3}$

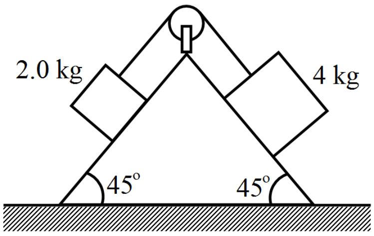
[Only magnitude has been taken]

Distance of the (A + P) from centre of mass $= \frac{l}{3}$ & for B it is $= \frac{2l}{3}$.

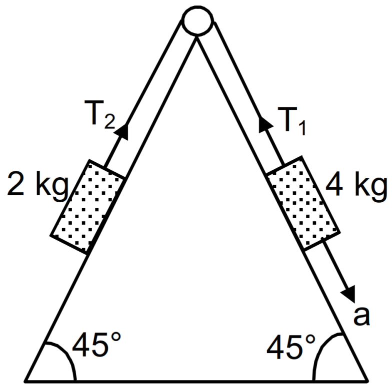
Therefore $P_{cm} = l_{cm} \times \omega$

$$\Rightarrow 2m \times \frac{v_0}{6} \times \frac{1}{3} + m \times \frac{v_0}{3} \times \frac{2l}{3}$$
$$= 2m\left(\frac{1}{3}\right)^2 + m\left(\frac{2l}{3}\right)^2 \times \omega$$
$$\Rightarrow \frac{6mv_0 l}{18} = \frac{6ml}{9} \times \omega$$
$$\Rightarrow \omega = \frac{v_0}{2l}$$

139. The pulley shown in figure has a radius 10cm and moment of inertia 0.5kg-m^2 about its axis. Assuming the inclined planes to be frictionless, calculate the acceleration of the 4.0kg block.



Ans. :



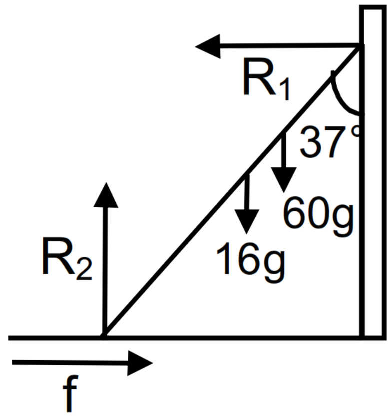
$$m_1 g \sin\theta - T_1 = m_1 a \; \ldots (1)$$
$$(T_1 - T_2) = \frac{la}{r^2} \; \ldots (2)$$
$$T_2 - m_2 g \sin\theta = m_2 a \; \ldots (3)$$

Adding the equations (1) and (3) we will get

$$m_1 g \sin\theta + (T_2 - T_1) - m_2 g \sin\theta = (m_1 + m_2)a$$
$$\Rightarrow (m_1 - m_2) g \sin\theta = \left(m_1 + m_2 + \frac{1}{r^2}\right)a$$
$$\Rightarrow a = \frac{(m_1 - m_2) g \sin\theta}{\left(m_1 + m_2 + \frac{1}{r^2}\right)} = 0.248 = 0.25 ms^{-2}$$

140. A uniform ladder of length 10.0m and mass 16.0kg is resting against a vertical wall making an angle of 37° with it. The vertical wall is frictionless but the ground is rough. An electrician weighing 60.0kg climbs up the ladder. If he stays on the ladder at a point 8.00m from the lower end, what will be the normal force and the force of friction on the ladder by the ground? What should be the minimum coefficient of friction for the elctrician to work safely?

Ans. :



$\mathrm{R_1 = \mu R_2,\ R_2 = 16g + 60g = 745N}$

$\mathrm{R_1 \times 10\cos 37^\circ = 16g \times 5\sin 37^\circ + 60g \times 8 \times \sin 37^\circ}$

$\Rightarrow \mathrm{8R_1 = 48g + 288g}$

$\Rightarrow \mathrm{R_1 = \frac{336g}{8} = 412N = f}$

Therefore $\mu = \frac{\mathrm{R_1}}{\mathrm{R_2}} = \frac{412}{745} = 0.53$

141. A ball is whirled in a circle by attaching it to a fixed point with a string. Is there an angular rotation of the ball about its centre? If yes, is this angular velocity equal to the angular velocity of the ball about the fixed point?

Ans. : Yes, there is an angular rotation of the ball about its centre.

Yes, angular velocity of the ball about its centre is same as the angular velocity of the ball about the fixed point.

Explanation:

Let the time period of angular rotation of the ball be T.

Therefore, we get:

Angular velocity of the ball about the fixed point $= \frac{2\pi}{\mathrm{T}}$

After one revolution about the fixed centre is completed, the ball has come back to its original position. In this case, the point at which the ball meets with the string is again visible after one revolution. This means that it has undertaken one complete rotation about its centre.

The ball has taken one complete rotation about its centre. Therefore, we have:

Angular displacement of the ball $= 2\pi$

Time period = T

So, angular velocity is again $\frac{2\pi}{\mathrm{T}}$. Thus, in both the cases, angular velocities are the same.

* Case study based questions [32]

142. Read the passage given below and answer the following questions from 1 to 5.

Moment of Inertia A heavy wheel called flywheel is attached to the shaft of steam engine, automobile engine etc., because of its large moment of inertia, the flywheel opposes the sudden increase or decrease of the speed of the vehicle. It allows a gradual change in the speed and prevents jerky motion and hence ensure smooth ride of passengers.

i. Moment of inertia of a body depends upon:
   a. axis of rotation
   b. torque
   c. angular momentum
   d. angular velocity
ii. A particle of mass 1 kg is kept at (1m, 1m, 1m). The moment of inertia of this particle about Z-axis would be:
   a. 1 kg-m^2
   b. 2 kg-m^2
   c. 3 kg-m^2
   d. (None of the above)
iii. Moment of inertia of a rod of mass m and length l about its one end is I. If one-fourth of its length is cut away, then moment of inertia of the remaining rod about its one end will be:
   a. $\frac{3}{4}\mathrm{I}$
   b. $\frac{9}{16}\mathrm{I}$
   c. $\frac{27}{64}\mathrm{I}$
   d. $\frac{\mathrm{I}}{16}$
iv. A circular disc is to be made by using iron and aluminium, so that it acquires maximum moment of inertia about its geometrical axis. It is possible with:
   a. iron and aluminium layers in alternate order
   b. aluminium at interior and iron surrounding it
   c. iron at interior and aluminium surrounding it
   d. Either (a) or (c)
v. Three thin rods each of length L and mass M are placed along X ,Y and Z -axes such that one end of each rod is at origin. The moment of inertia of this system about Z-axis is:
   a. $\frac{2}{3}\mathrm{ML}^2$
   b. $\frac{4\mathrm{ML}^2}{3}$
   c. $\frac{5\mathrm{ML}^2}{3}$
   d. $\frac{\mathrm{ML}^2}{3}$

Ans. :

i. (a) axis of rotation

Explanation:

Moment of inertia of a body depends on position and orientation of the axis of rotation with respect to the body.

ii. (b) 2 kg-m^2

Explanation:

Perpendicular distance from Z-axis would be

$\sqrt{(1)^2+(1)^2}=\sqrt{2\mathrm{m}}$

$\therefore\ \mathrm{I}=\mathrm{Mr}^2=(1)(\sqrt{2})^2=2\mathrm{kg}-\mathrm{m}^2$

iii. c $\frac{27}{64}\mathrm{I}$

Explanation:

Initial moment of inertia, $\mathrm{I}=\frac{\mathrm{ml}^2}{3}$

New moment of inertia,

$$\mathrm{I}=\frac{\left(\frac{3\mathrm{m}}{4}\right)\left(\frac{3\mathrm{l}}{4}\right)^2}{3}=\frac{27}{64}\left(\frac{\mathrm{ml}^2}{3}\right)=\frac{27}{64}\mathrm{I}$$

iv. (b) aluminium at interior and iron surrounding it

Explanation:

A circular disc is made up of larger number of circular rings. Moment of inertia of a circular ring in given by

$$\mathrm{I}=\mathrm{MR}^2$$

$$\Rightarrow \mathrm{I}\infty\mathrm{M}$$

Since, mass is proportional to the density of material. The density of iron is more than that of aluminium. Hence to get maximum value of I , the less dense material should be used at interior and denser at the surrounding. Therefore, using aluminium at the interior and iron at its surrounding will maximise the moment of inertia.

v. $\frac{2}{3}\mathrm{ML}^2$

Explanation:

Moment of inertia of the rod lying along Z-axis will be zero. Moment of inertia of the rods along X and Y -axes will be $\frac{\mathrm{ML}^2}{3}$ each.

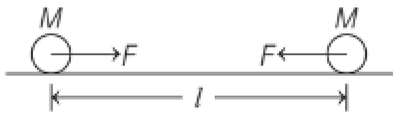
143. Read the passage given below and answer the following questions from 1 to 5. **Centre of Mass:**

The centre of mass of a body or a system of bodies is the point which moves as though all of the mass were concentrated there and all external forces were applied to it. Hence, a point at which the entire mass of the body or system of bodies is supposed to be concentrated is known as the centre of mass. If a system consists of more than one particles (or bodies) and net external force on the system in a particular direction is zero with centre of mass at rest. Then, the centre of mass will not move along that direction. Even though some particles of the system may move along that direction.

i. The centre of mass of a system of two particles divides, the distance between them:
   a. in inverse ratio of square of masses of particles
   b. in direct ratio of square of masses of particles
   c. in inverse ratio of masses of particles
   d. in direct ratio of masses of particles

ii. Two bodies of masses 1kg and 2 kg are lying in xy-plane at (-1, 2) and (2, 4) respectively. What are the coordinates of the centre of mass?
   a. $\left(1,\frac{10}{3}\right)$
   b. (1, 10)
   c. (0, 1)
   d. None of these

iii. Two balls of same masses start moving towards each other due to gravitational attraction, if the initial distance between them is l. Then, they meet at:



a. $\frac{l}{2}$
b. l
c. $\frac{l}{3}$
d. $\frac{l}{4}$

iv. All the particles of a body are situated at a distance R from the origin. The distance of centre of mass of the body from the origin is:
a. $= \mathrm{R}$
b. $\leq \mathrm{R}$
c. $> \mathrm{R}$
d. $\geq \mathrm{R}$

v. Two particles A and B initially at rest move towards each other under a mutual force of attraction. At the instant, when the speed of A is v and the speed of B is 2v, the speed of centre of mass of the system is:
a. zero
b. v
c. 1. 5v
d. 3v

Ans. :

i. (c) in inverse ratio of masses of particles

Explanation:

Centre of mass of a system of two particles is

Then, $\mathrm{r_{cm}} = \frac{\mathrm{m_1 r_1 + m_2 r_2}}{\mathrm{M}}$

If $m_1 + m_2$ = M = total mass of the particles,

then $\mathrm{r_{cm}} = \frac{\mathrm{m_1 r_1 + m_2 r_2}}{\mathrm{M}}$

$\therefore\ \mathrm{r_{cm}} \infty \frac{1}{\mathrm{M}}$

So, the above relation clearly shows that the centre of mass of a system of two particles divide the distance between them in inverse ratio of masses of particles.

ii. $\left(1, \frac{10}{3}\right)$

Explanation:

Let the coordinates of the centre of mass be (x, y).

$\therefore\ \mathrm{x} = \frac{\mathrm{m_1 x_1 + m_2 x_2}}{\mathrm{m_1 + m_2}}$

$\frac{1 \times (-1) + 2 \times 2}{3} = \frac{-1+4}{3} = 1$

$y = \frac{\mathrm{m_1 y_1 + m_2 y_2}}{\mathrm{m_1 + m_2}}$

$= \frac{1 \times 2 + 2 \times 4}{3} = \frac{2+8}{3} = \frac{10}{3}$

iii. $\frac{1}{2}$

Explanation:

As the balls were initially at rest and the forces of attraction are internal, then their centre of mass (CM) will always remain at rest.

So, $v_{cm} = 0$

As CM is at rest, they will meet at CM.

iv. $\leq \mathrm{R}$

Explanation:

For a single particle, distance of centre of mass from origin is R. For more than one particles, distance $\leq \mathrm{R}$

v. zero

Explanation:

As per the question, two particles A and B are initially at rest, move towards each other under a mutual force of attraction. It means that, no external force is applied on the system. Therefore, $F_{ext} = 0$.

So, there is no acceleration of CM. This means velocity of the CM remain constant.

As, initial velocity of CM, $v_i = 0$ and final velocity of CM, $v_f = 0$.

144. A kid of mass M stands at the edge of a platform of radius R which can be freely rotated about its axis. The moment of inertia of the platform is I. The system is at rest when a friend throws a ball of mass m and the kid catches it. If the velocity of the ball is v horizontally along the tangent to the edge of the platform when it was caught by the kid, find the angular speed of the platform after the event.

Ans. : A kid of mass M stands at the edge of a platform of radius R which has a moment of inertia I. A ball of m thrown to him and horizontal velocity of the ball v when he catches it.

Therefore if we take the total bodies as a system

Therefore $mvR = \{I + (M + m)R^2\}\omega$

(The moment of inertia of the kid and ball about the axis = $(M + m)R^2$)

$\Rightarrow \omega = \frac{\mathrm{mvR}}{1+(\mathrm{M}+\mathrm{m})\mathrm{R}^2}$

145. Mr. Verma (50kg) and Mr. Mathur (60kg) are sitting at the two extremes of a 4m long boat (40kg) standing still in water. To discuss a mechanics problem, they come to the middle of the boat. Neglecting friction with water, how far does the boat move on the water during the process?

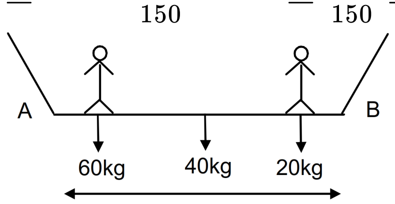
Ans. : $m_1 = 60kg$, $m_2 = 40kg$, $m_3 = 50kg$,

Let A be the origin of the system.

Initially Mr. Verma & Mr. Mathur are at extreme position of the boat.

$\therefore$ The centre of mass will be at a distance

$= \frac{60\times0+40\times2+50\times4}{150} = \frac{280}{150} = 1.87\mathrm{m}$ from ‘A’



When they come to the mid point of the boat the CM lies at 2m from ‘A’.

$\therefore$ The shift in CM = 2 - 1.87 = 0.13m towards right.

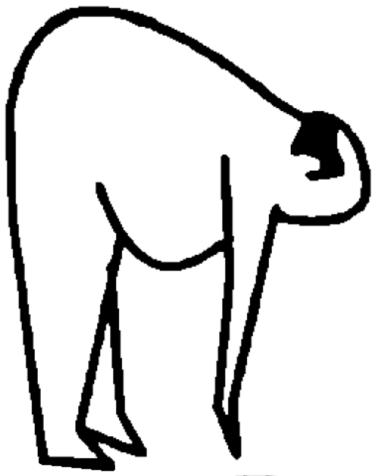
But, as there is no external force in longitudinal direction their CM would not shift.

So, the boat moves 0.13m or 13cm towards right.

146. What can be said about the centre of mass of a uniform hemisphere without making any calculation? Will its distance from the centre be more than $\frac{r}{2}$ or less than $\frac{r}{2}$?

Ans. : It would be less than $\frac{r}{2}$ as more of the mass is concentrated near center of the sphere so center of mass should be less than $\frac{r}{2}$.

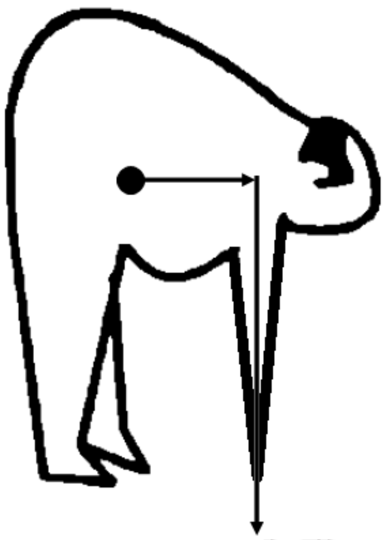
147. When a fat person tries to touch his toes, keeping the legs straight, he generally falls.



Explain with reference to figure.

Ans. :

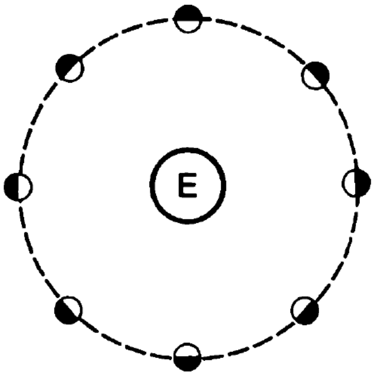
When the man tries to touch his toe, he exerts force along the hand downwards. This force produces amoment along the Centre of Mass CM of the man as shown in the figure.
This moment makes him rotateand, thus, he falls down after losing the balance.



148. When tall buildings are constructed on earth, the duration of day-night slightly increases. Is it true?

Ans. : Yes, because tall buildings have their CM much above the ground. It increases moment of inertia of the Earth. As the Earth’s rotation does not involve torque, its angular momentum is constant. Thus, an increase in MI leads to lower angular velocity of the Earth about its axis of rotation. This means length of night and day will increase. However, the increase is very small.

149. The moon rotates about the earth in such a way that only one hemisphere of the moon faces the earth. Can we ever see the ''other face'' of the moon from the earth? Can a person on the moon ever see all the faces of the earth?



Ans. : No, we cannot see the other face of the Moon from the Earth.

Yes, a person on the Moon can see all the faces of the Earth.

Explanation:

Angular velocity of the Moon about its own axis of rotation is same as its angular velocity of revolution about the Earth. This means that its rotation time period equals its revolution time period. So, we can see only one face of the Moon from the Earth.

However, angular velocity of the Earth about its axis is not same as the angular velocity of Moon about the Earth. So, all the faces of the Earth is visible from the Moon.

----- Know what sparks the light in you. Then use that light to illuminate the world.” -----